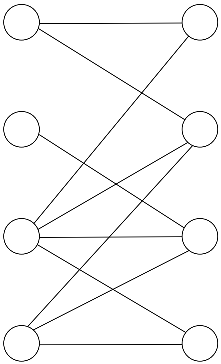
Testing Bipartiteness: Applications of BFS

Def. A graph $G = (V, E)$ is a bipartite graph if there is a partition of V into two sets L and R such that for every edge $(u, v) \in E$, either $u \in L, v \in R$ or $v \in L, u \in R$.



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- Report "not a bipartite graph" if contradiction was found

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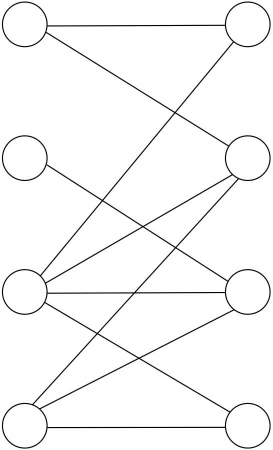
- Report "not a bipartite graph" if contradiction was found
- If G contains multiple connected components, repeat above algorithm for each component

Test Bipartiteness



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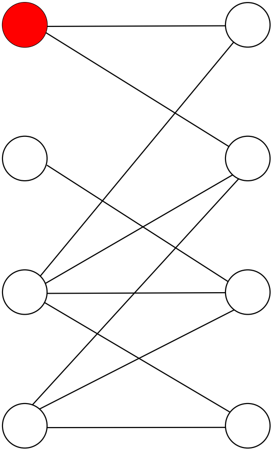

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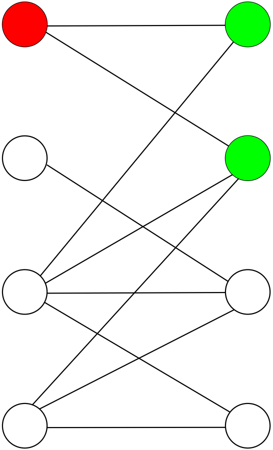

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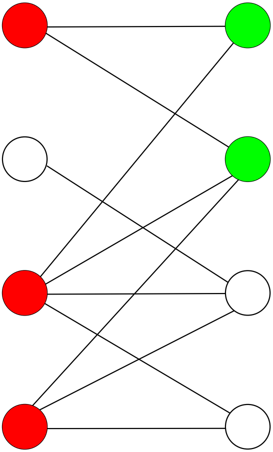

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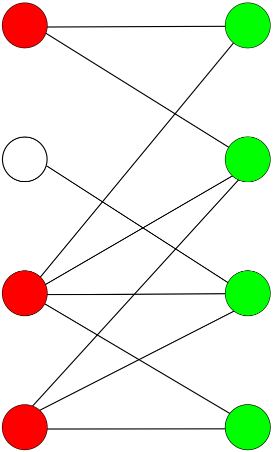

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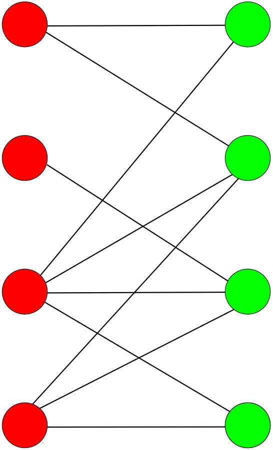

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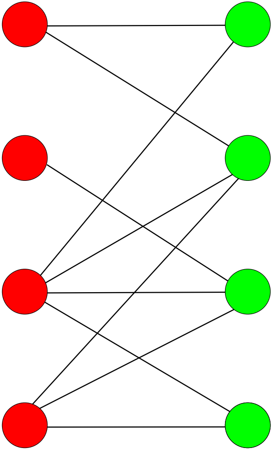

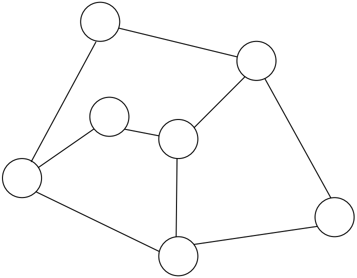

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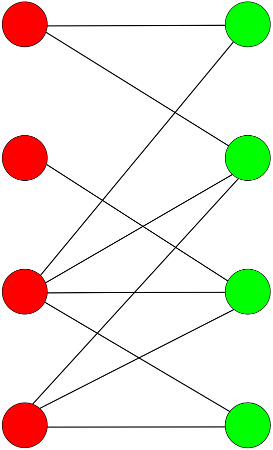

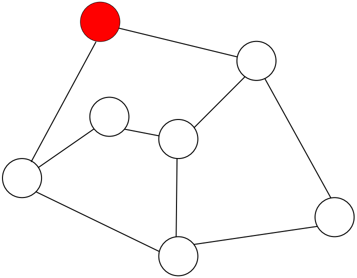

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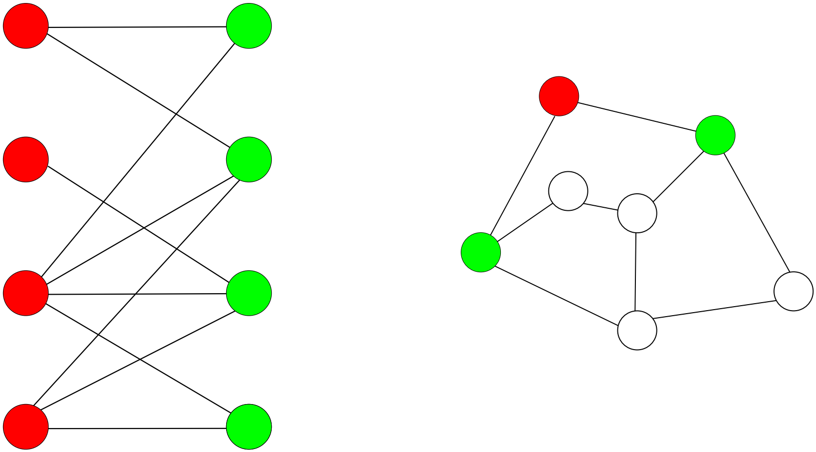

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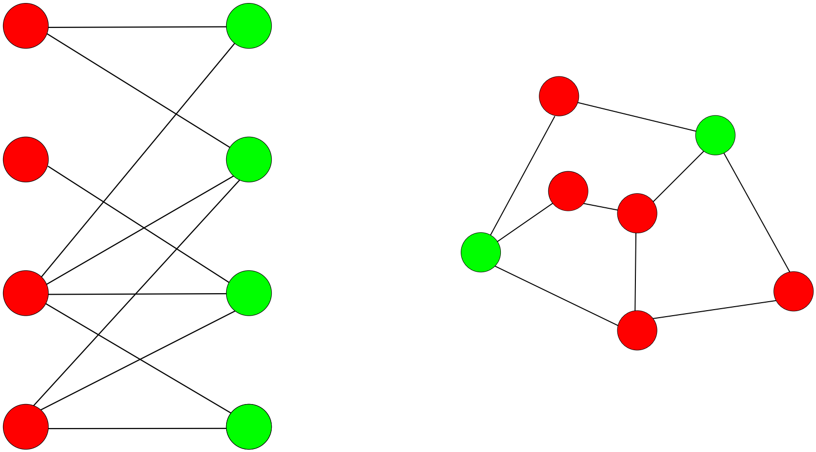

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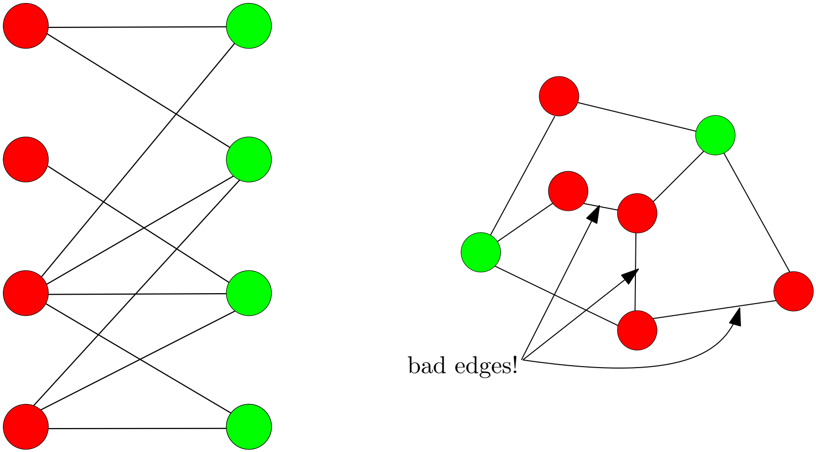

Testing Bipartiteness using BFS

BFS(s)

$head \leftarrow 1, tail \leftarrow 1, queue[1] \leftarrow s$
mark s as "visited" and all other vertices as "unvisited"
while $head \leq tail$ **do**
    $v \leftarrow queue[head], head \leftarrow head + 1$
    for all neighbors u of v **do**
        if u is "unvisited" **then**
            $tail \leftarrow tail + 1, queue[tail] = u$
            mark u as "visited"

Testing Bipartiteness using BFS

test-bipartiteness(s)

$head \leftarrow 1, tail \leftarrow 1, queue[1] \leftarrow s$
mark s as "visited" and all other vertices as "unvisited"
$color[s] \leftarrow 0$
while $head \leq tail$ **do**
    $v \leftarrow queue[head], head \leftarrow head + 1$
    for all neighbors u of v **do**
        if u is "unvisited" **then**
            $tail \leftarrow tail + 1, queue[tail] = u$
            mark u as "visited"
            $color[u] \leftarrow 1 - color[v]$
        else if $color[u] = color[v]$ **then**
            print("G is not bipartite") and exit

Testing Bipartiteness using BFS

mark all vertices as "unvisited"
for each vertex $v \in V$ **do**
    if v is "unvisited" **then**
        test-bipartiteness(v)
print("G is bipartite")

Testing Bipartiteness using BFS

1: mark all vertices as "unvisited"
2: **for** each vertex $v \in V$ **do**
3:     **if** v is "unvisited" **then**
4:         test-bipartiteness(v)
5: print("G is bipartite")

Obs. Running time of algorithm $= O(n + m)$

Testing Bipartiteness using DFS

test-bipartiteness-DFS(s)

1: mark all vertices as "unvisited"
2: recursive-test-DFS(s)

recursive-test-DFS(v)

1: mark v as "visited"
2: **for** all neighbors u of v **do**
3:     **if** u is unvisited **then** , recursive-test-DFS(u)

Testing Bipartiteness using DFS

test-bipartiteness-DFS(s)

```
mark all vertices as "unvisited"
color[s] ← 0
recursive-test-DFS(s)
```

recursive-test-DFS(v)

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mark v as "visited"
for all neighbors u of v do
    if u is unvisited then
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    else if color[u] = color[v] then
        print("G is not bipartite") and exit
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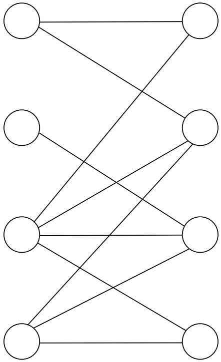
Bipartite Graph

Def. An undirected graph $G = (V, E)$ is a bipartite graph if there is a partition of V into two sets L and R such that for every edge $(u, v) \in E$, either $u \in L, v \in R$ or $v \in L, u \in R$.



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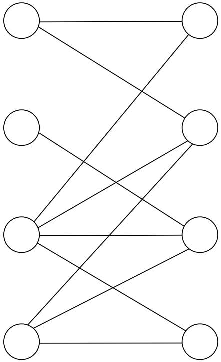
Obs. Bipartite graph may contain cycles.



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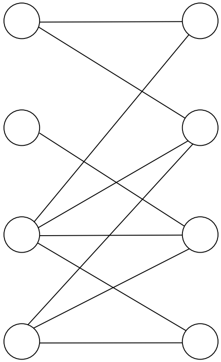
Obs. If a graph is a tree, then it is also a bipartite graph.



BFS and DFS

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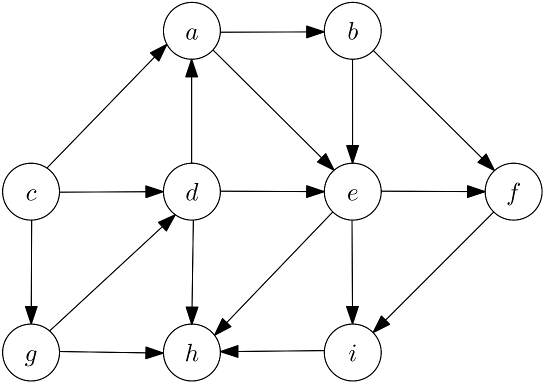
Outline

Topological Ordering Problem

Input: a directed acyclic graph (DAG) $G = (V, E)$

Output: 1-to-1 function $\pi : V \to \{1, 2, 3 \cdots, n\}$, so that

- if $(u, v) \in E$ then $\pi(u) < \pi(v)$



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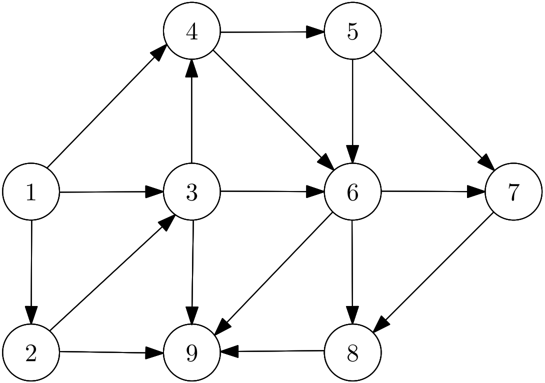
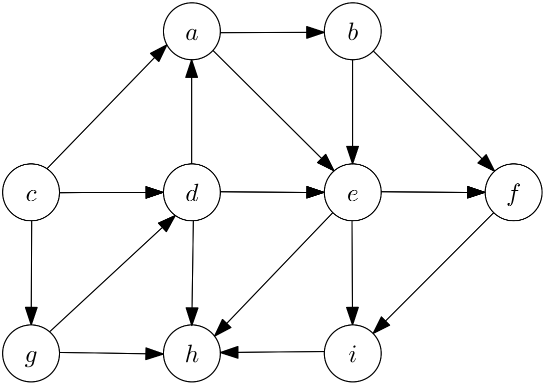


Topological Ordering

- Algorithm: each time take a vertex without incoming edges, then remove the vertex and all its outgoing edges.



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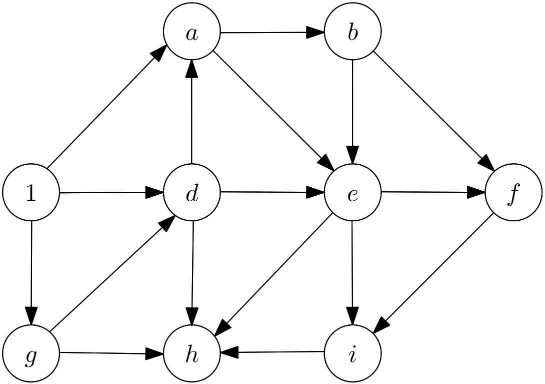

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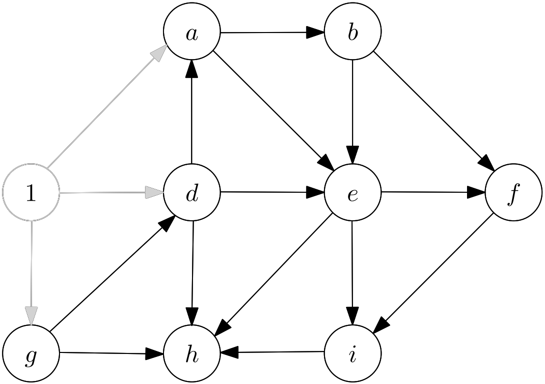

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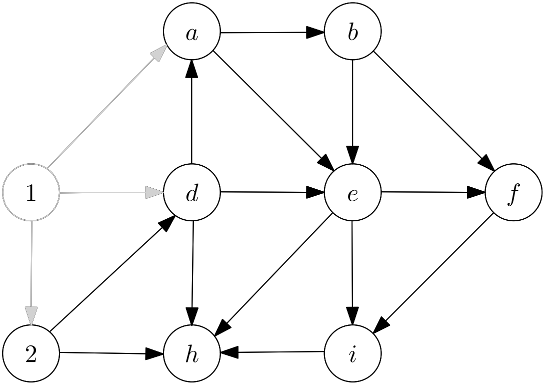

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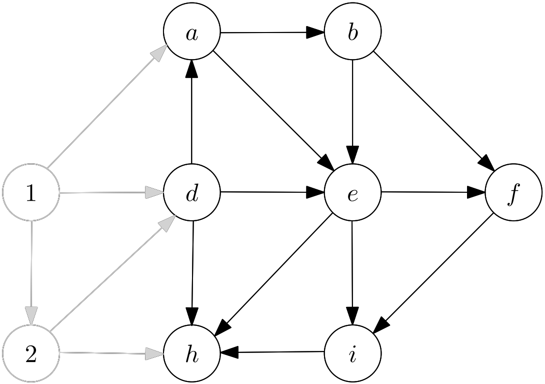

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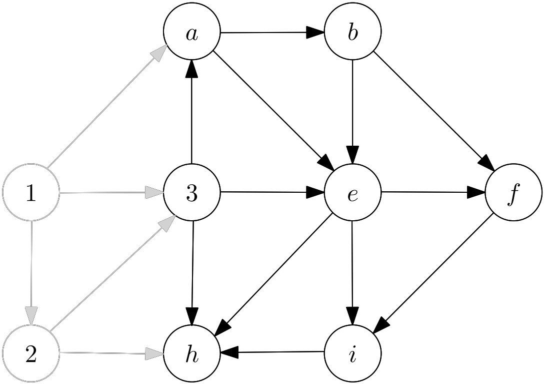

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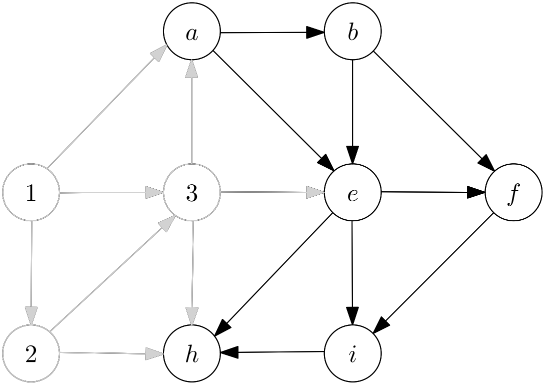

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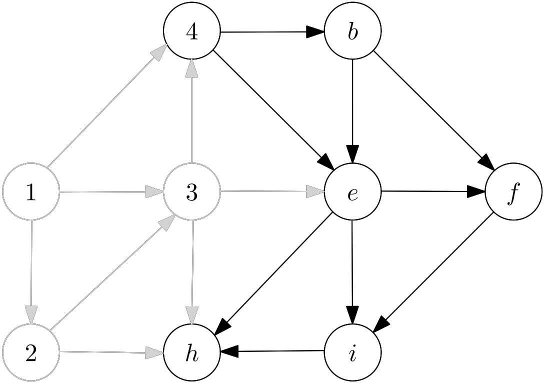

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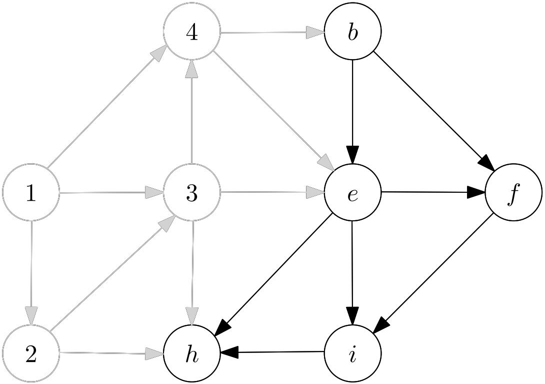

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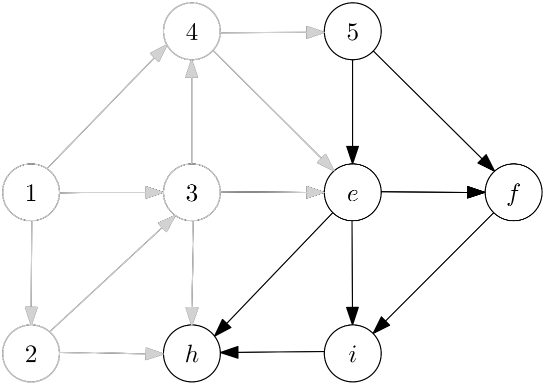

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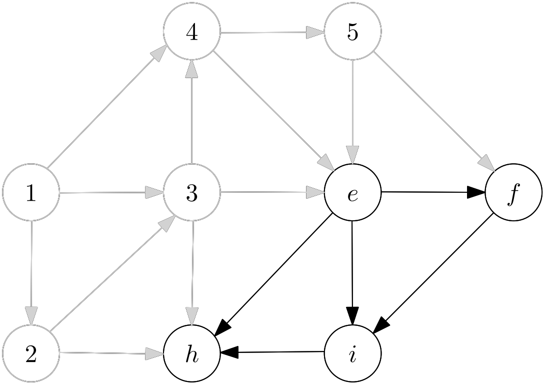

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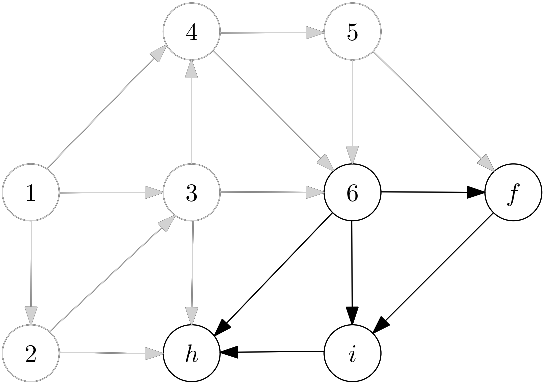

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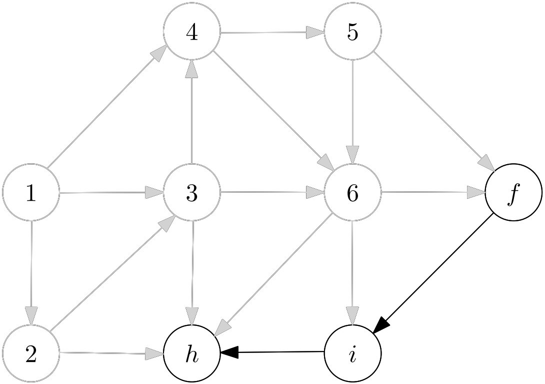

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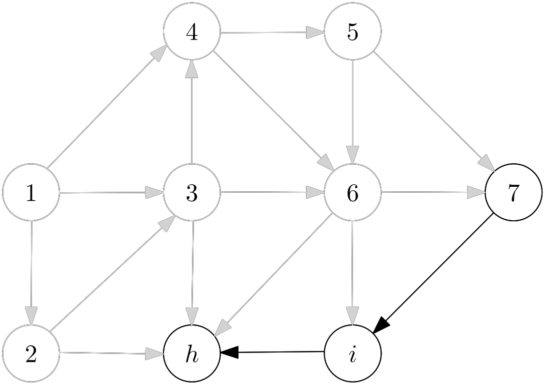

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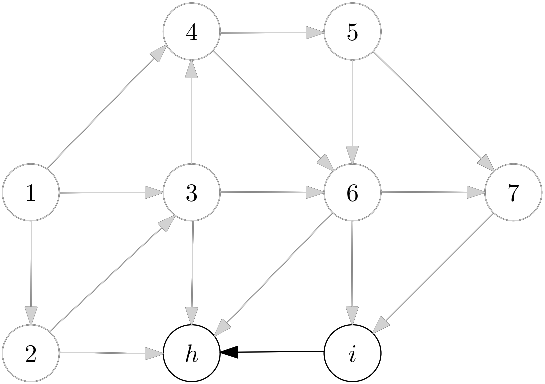

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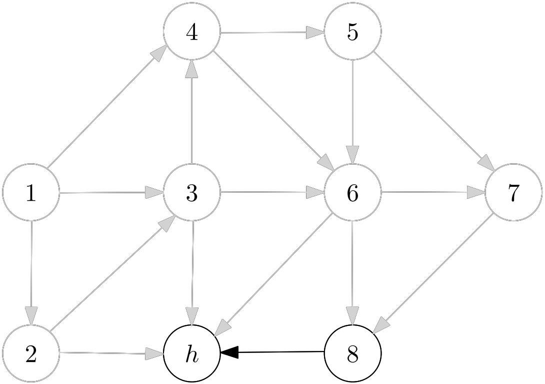

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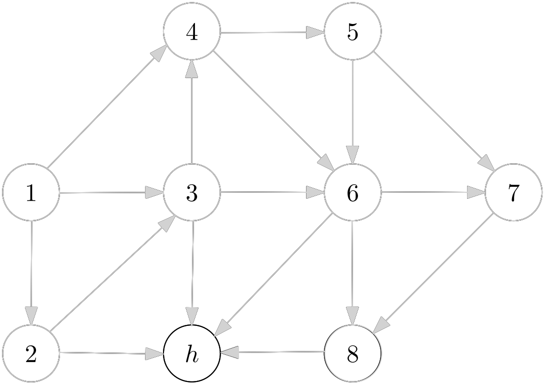

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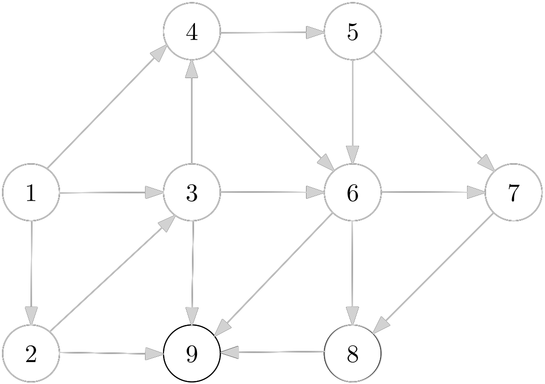

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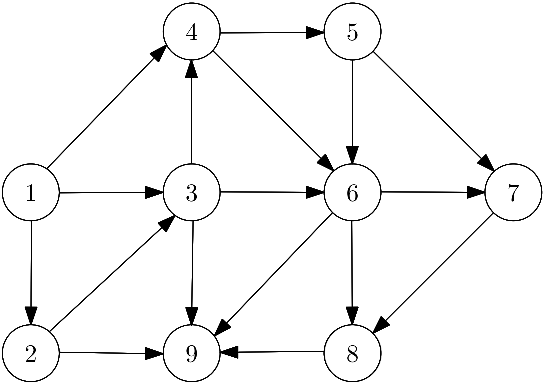

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Q: How to make the algorithm as efficient as possible?

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Q: How to make the algorithm as efficient as possible?

A:

- Use linked-lists of outgoing edges
- Maintain the in-degree d_v of vertices
- Maintain a queue (or stack) of vertices v with $d_v = 0$

topological-sort(G)

```
let d_v ← 0 for every v ∈ V
for every v ∈ V do
    for every u such that (v, u) ∈ E do
        d_u ← d_u + 1
S ← {v : d_v = 0}, i ← 0
while S ≠ ∅ do
    v ← arbitrary vertex in S, S ← S \ {v}
    i ← i + 1, π(v) ← i
    for every u such that (v, u) ∈ E do
        d_u ← d_u − 1
        if d_u = 0 then add u to S
if i < n then output "not a DAG"
```

- S can be represented using a queue or a stack
- Running time $= O(n + m)$