Connectivity Problem

Input: graph $G = (V, E)$, (using linked lists)

two vertices $s, t \in V$

Output: whether there is a path connecting s to t in G

- Algorithm: starting from s, search for all vertices that are reachable from s and check if the set contains t
  - Breadth-First Search (BFS)
  - Depth-First Search (DFS)

Breadth-First Search (BFS)

- Build layers $L_0, L_1, L_2, L_3, \cdots$
- $L_0 = \{s\}$
- L_{j+1} contains all nodes that are not in $L_0 \cup L_1 \cup \cdots \cup L_j$ and have an edge to a vertex in L_j

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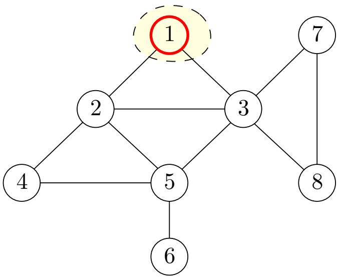

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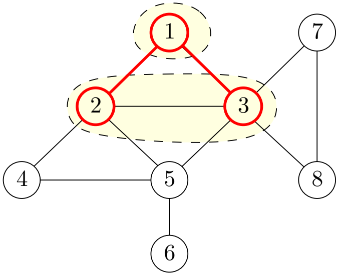

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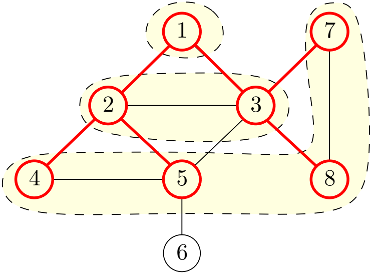

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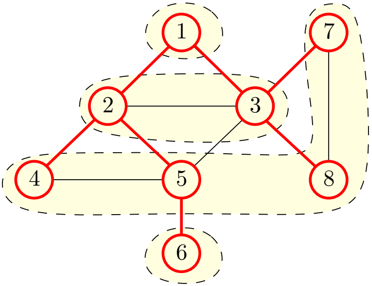


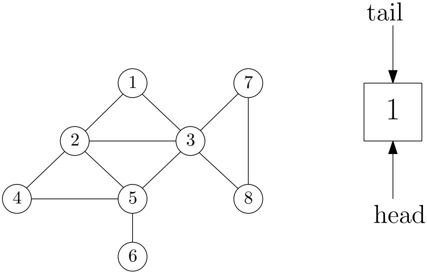
Implementing BFS using a Queue

BFS(s)

```
head ← 1, tail ← 1, queue[1] ← s
mark s as "visited" and all other vertices as "unvisited"
while head ≤ tail do
    v ← queue[head], head ← head + 1
    for all neighbors u of v do
        if u is "unvisited" then
            tail ← tail + 1, queue[tail] = u
            mark u as "visited"
```

- Running time: $O(n+m)$.

Example of BFS via Queue



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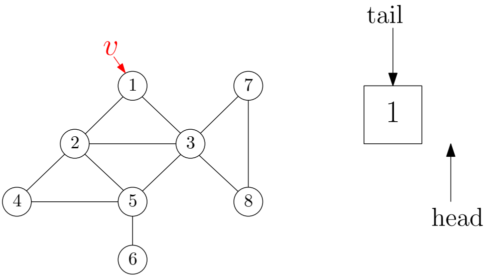

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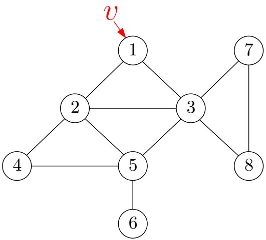

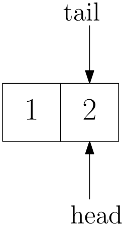

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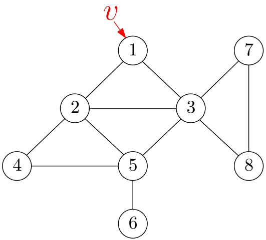

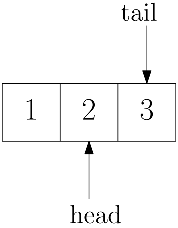

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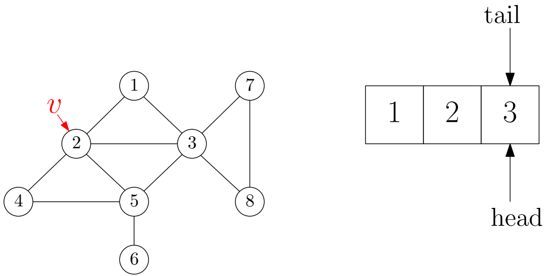

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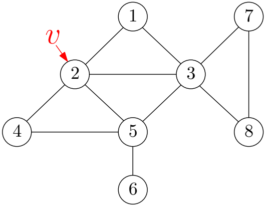

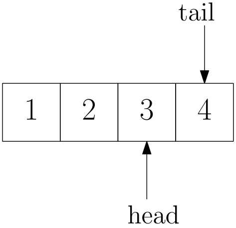

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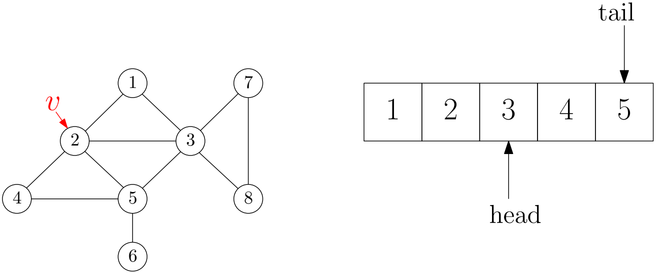

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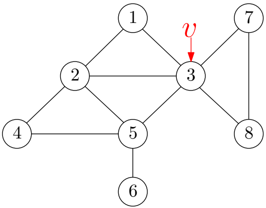

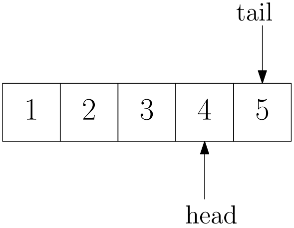

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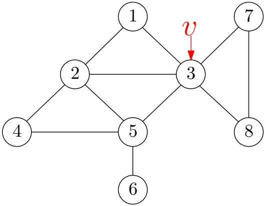

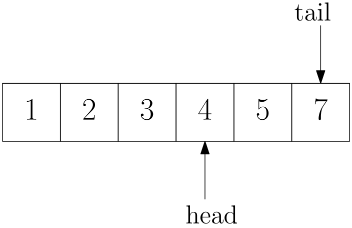

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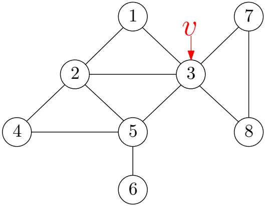

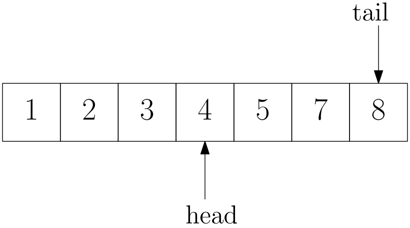

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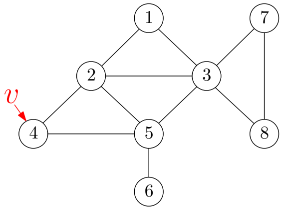
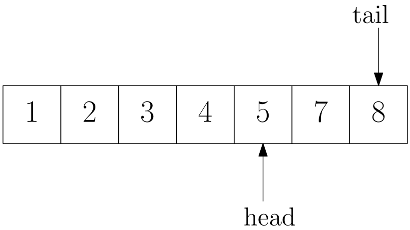
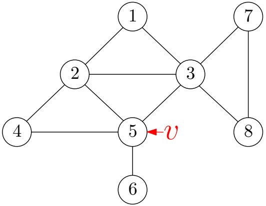
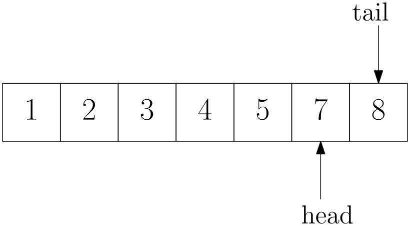





1
2
3
4
5
6
7
8
v


tail
1
2
3
4
5
7
8
head

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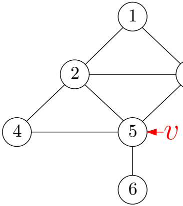

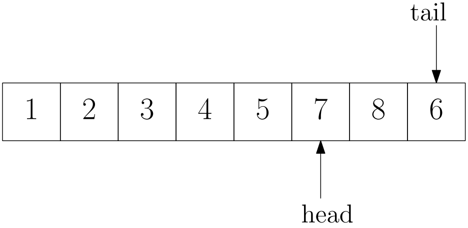

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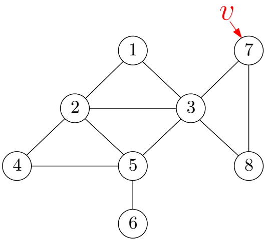

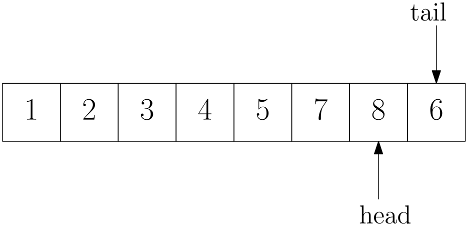

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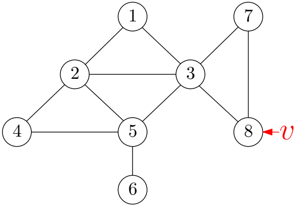

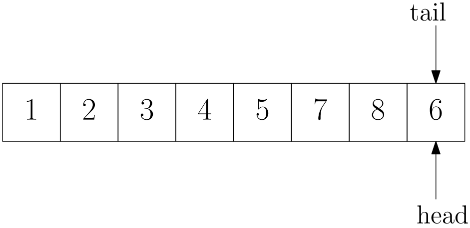

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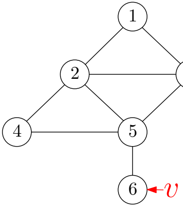

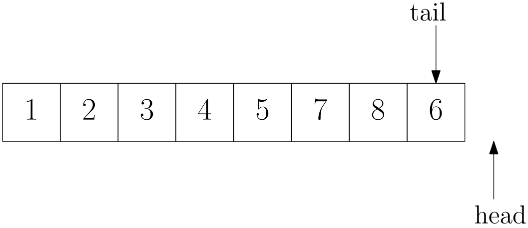

Depth-First Search (DFS)

- Starting from s
- Travel through the first edge leading out of the current vertex
- When reach an already-visited vertex ("dead-end"), go back
- Travel through the next edge
- If tried all edges leading out of the current vertex, go back

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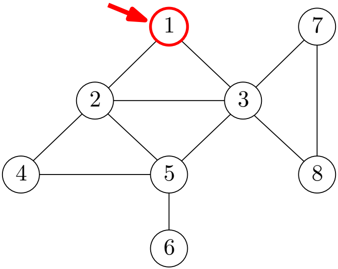

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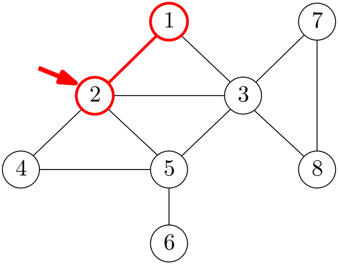

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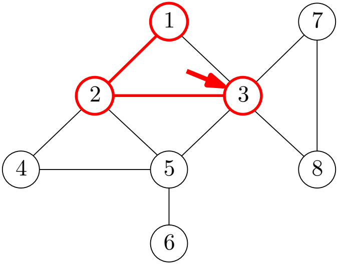

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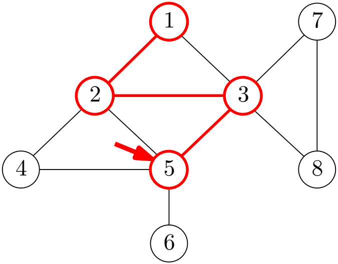

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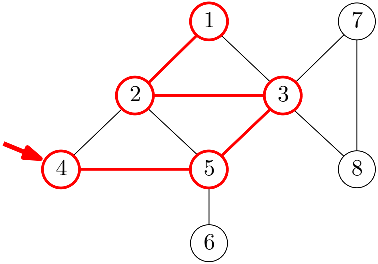

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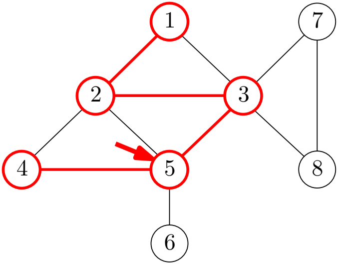

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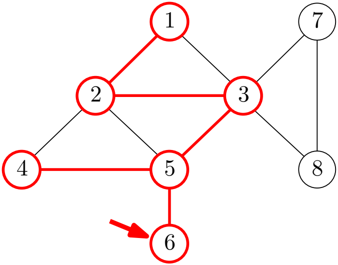

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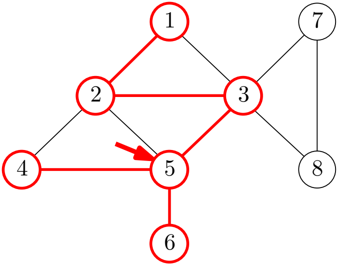

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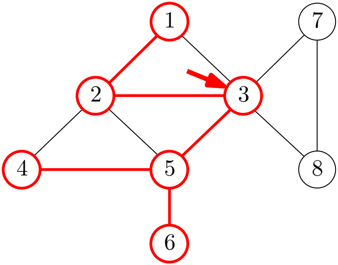

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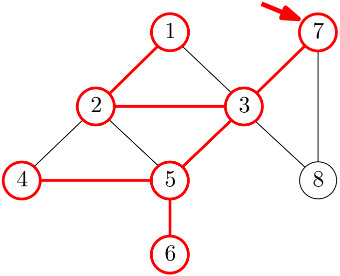

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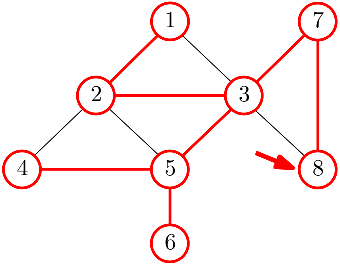


Implementing DFS using Recurrsion

DFS(s)

```
mark all vertices as "unvisited"
recursive-DFS(s)
```

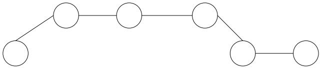
recursive-DFS(v)

```
mark v as "visited"
for all neighbors u of v do
    if u is unvisited then recursive-DFS(u)
```

Outline

Path Graph (or Linear Graph)

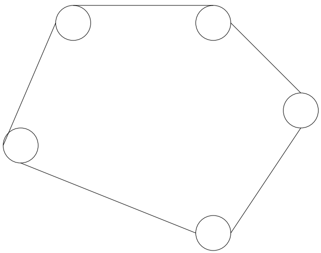
Def. An undirected graph $G = (V, E)$ is a path if the vertices can be listed in an order $\{v_1, v_2, ..., v_n\}$ such that the edges are the $\{v_i, v_{i+1}\}$ where $i = 1, 2, ..., n - 1$.



- Path graphs are connected graphs.

Cycle Graph (or Circular Graph)

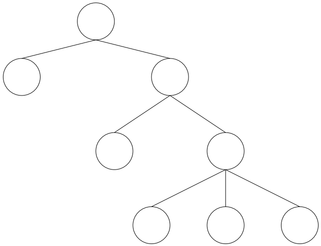
Def. An undirected graph $G = (V, E)$ is a cycle if its vertices can be listed in an order $v_1, v_2, ..., v_n$ such that the edges are the $\{v_i, v_{i+1}\}$ where $i = 1, 2, ..., n-1$, plus the edge $\{v_n, v_1\}$.



- The degree of all vertices is 2.

Tree

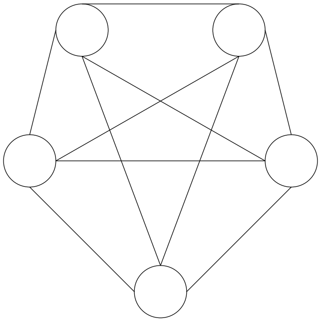
Def. An undirected graph $G = (V, E)$ is a tree if any two vertices are connected by exactly one path. Or the graph is a connected acyclic graph.



- Most important type of special graphs: most computational problems are easier to solve on trees or lines.

Complete Graph

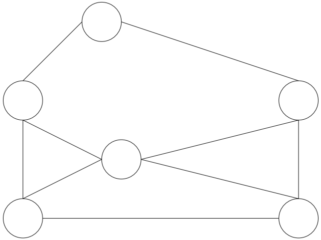
Def. An undirected graph $G = (V, E)$ is a complete graph if each pair of vertices is joined by an edge.



- A complete graph contains all possible edges.

Planar Graph

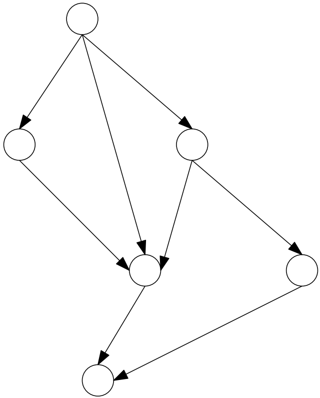
Def. An undirected graph $G = (V, E)$ is a planar graph if its vertices and edges can be drawn in a plane such that no two of the edges intersect.



- Most computational problems have good solutions in a planar graph.

Directed Acyclic Graph (DAG)

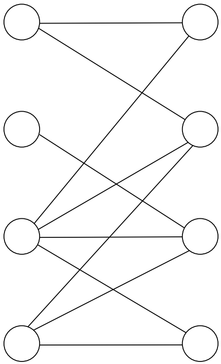
Def. A directed graph $G = (V, E)$ is a directed acyclic graph if it is a directed graph with no directed cycles



- DAG is equivalent to a partial ordering of nodes.

Bipartite Graph

Def. An undirected graph $G = (V, E)$ is a bipartite graph if there is a partition of V into two sets L and R such that for every edge $(u, v) \in E$, either $u \in L, v \in R$ or $v \in L, u \in R$.

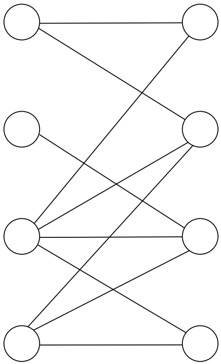


Outline

Outline

Testing Bipartiteness: Applications of BFS

Def. A graph $G = (V, E)$ is a bipartite graph if there is a partition of V into two sets L and R such that for every edge $(u, v) \in E$, either $u \in L, v \in R$ or $v \in L, u \in R$.



Testing Bipartiteness

- Taking an arbitrary vertex $s \in V$

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- Assuming $s \in L$ w.l.o.g

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Testing Bipartiteness

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Testing Bipartiteness

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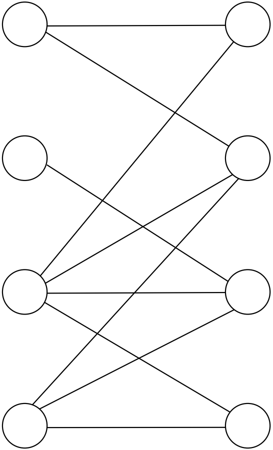
- $\cdots$
- Report "not a bipartite graph" if contradiction was found

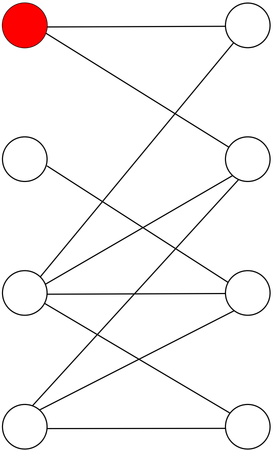
Testing Bipartiteness

- Taking an arbitrary vertex $s \in V$
- Assuming $s \in L$ w.l.o.g
- Neighbors of s must be in R
- Neighbors of neighbors of s must be in L
- $\cdots$
- Report "not a bipartite graph" if contradiction was found
- If G contains multiple connected components, repeat above algorithm for each component

Test Bipartiteness



Test Bipartiteness



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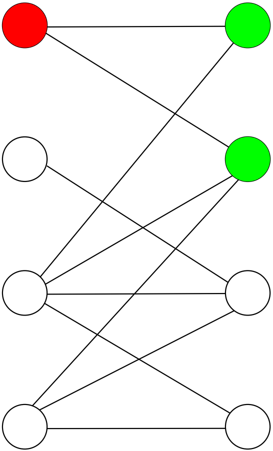

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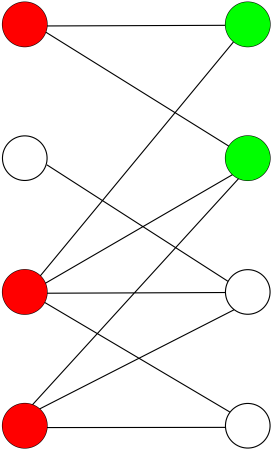

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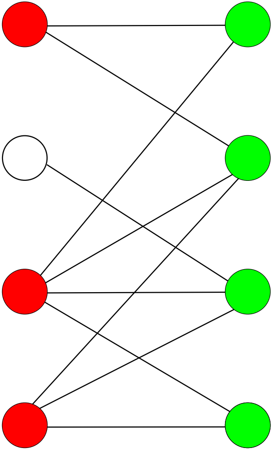

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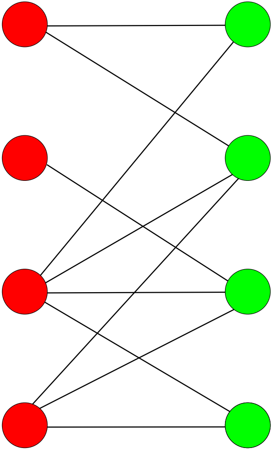

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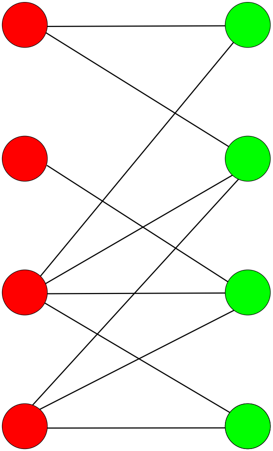

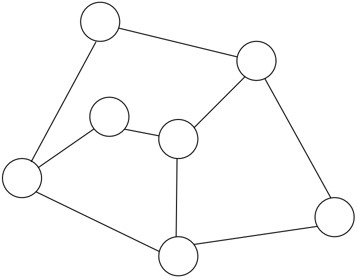

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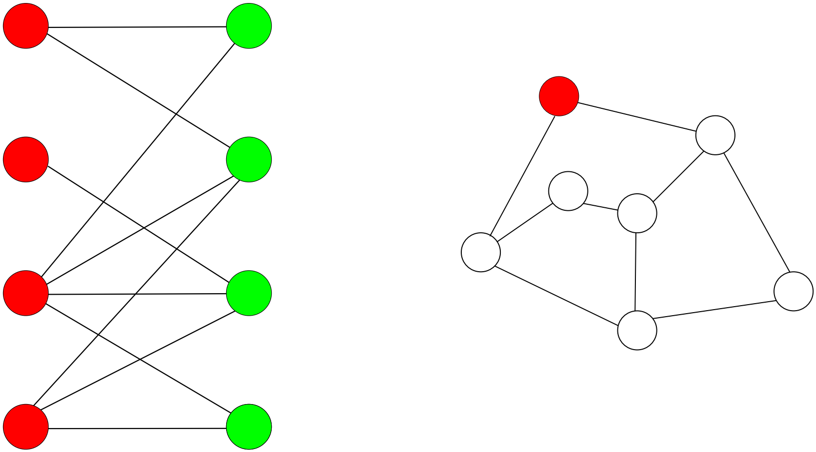

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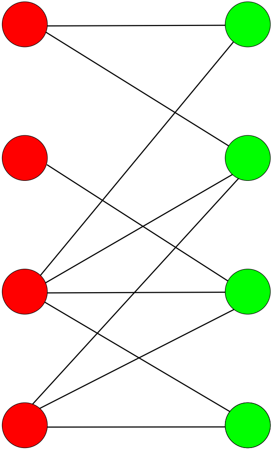

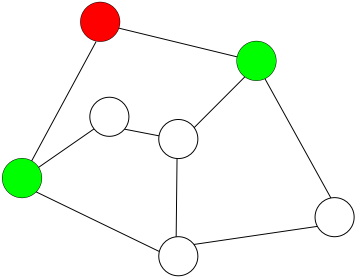

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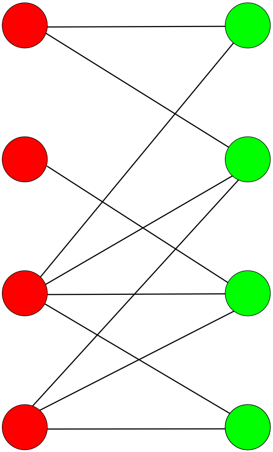

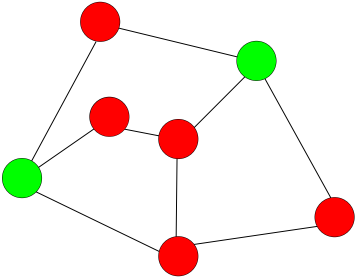

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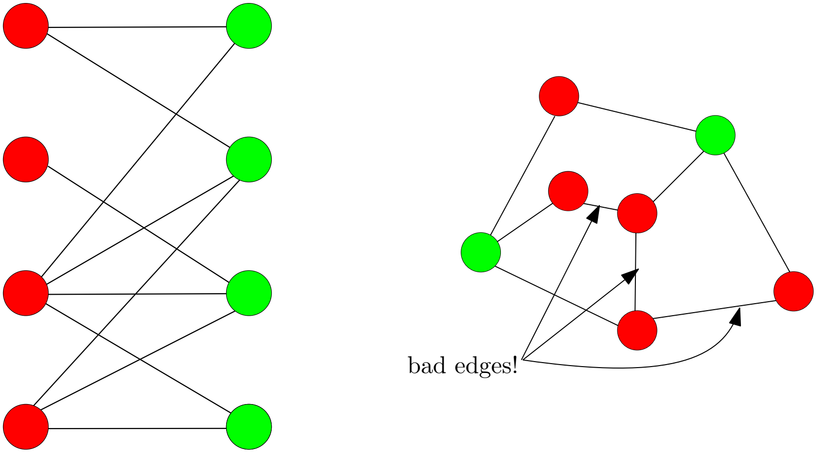

Testing Bipartiteness using BFS

BFS(s)

$head \leftarrow 1, tail \leftarrow 1, queue[1] \leftarrow s$
mark s as "visited" and all other vertices as "unvisited"
while $head \leq tail$ **do**
    $v \leftarrow queue[head], head \leftarrow head + 1$
    for all neighbors u of v **do**
        if u is "unvisited" **then**
            $tail \leftarrow tail + 1, queue[tail] = u$
            mark u as "visited"

Testing Bipartiteness using BFS

test-bipartiteness(s)

$head \leftarrow 1, tail \leftarrow 1, queue[1] \leftarrow s$
mark s as "visited" and all other vertices as "unvisited"
$color[s] \leftarrow 0$
while $head \leq tail$ **do**
    $v \leftarrow queue[head], head \leftarrow head + 1$
    for all neighbors u of v **do**
        if u is "unvisited" **then**
            $tail \leftarrow tail + 1, queue[tail] = u$
            mark u as "visited"
            $color[u] \leftarrow 1 - color[v]$
        else if $color[u] = color[v]$ **then**
            print("G is not bipartite") and exit