

CSE 431/531: Algorithm Analysis and Design (Fall 2023)

Graph Algorithms

Lecturer: Kelin Luo

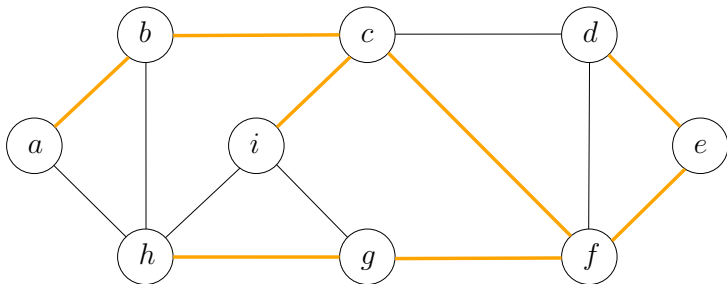
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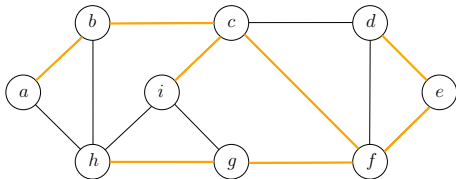
Outline

- 1 Minimum Spanning Tree
 - Kruskal's Algorithm
 - Reverse-Kruskal's Algorithm
 - Prim's Algorithm
- 2 Single Source Shortest Paths
 - Dijkstra's Algorithm
- 3 Shortest Paths in Graphs with Negative Weights
- 4 All-Pair Shortest Paths and Floyd-Warshall

Spanning Tree

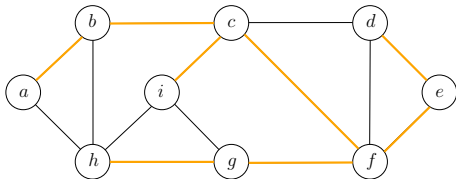
Def. Given a connected graph $G = (V, E)$, a **spanning tree** $T = (V, F)$ of G is a sub-graph of G that is a tree including all vertices V .





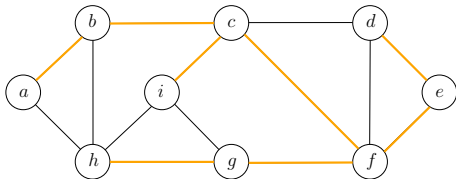
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- T is a spanning tree of G ;



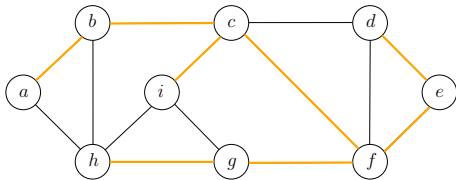
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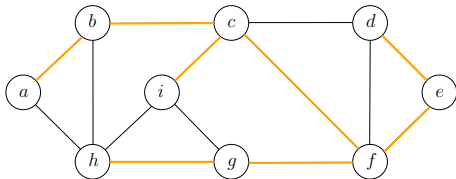
Lemma Let $T = (V, F)$ be a subgraph of $G = (V, E)$. The following statements are equivalent:

- T is a spanning tree of G ;
- T is acyclic and connected;
- T is connected and has $n - 1$ edges;



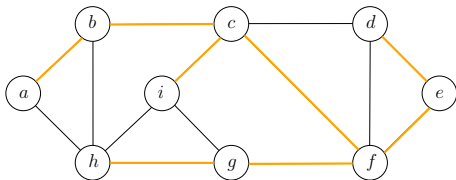
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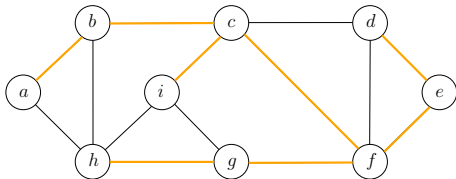
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- T is minimally connected: removal of any edge disconnects it;
- T is maximally acyclic: addition of any edge creates a cycle;
- T has a unique simple path between every pair of nodes.

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 - BFS
 - DFS

Minimum Spanning Tree (MST) Problem

Input: Graph $G = (V, E)$ and edge weights $w : E \rightarrow \mathbb{R}$

Output: the spanning tree T of G with the minimum total weight