Why do PIC Simulations Get Noisy?

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SIAM CSE 2025
Fort Worth, TX
Mar 7, 2025

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Never believe anything

until you run it.

Why do PIC simulations eventually become noisy?

```
cd $PETSC_DIR/src/ts/tutorials/hamiltonian
make ex2
./ex2 -options_file siam.opts
```

`siam.opts`: pastebin.com/wVcMhQ15

Vlasov

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f)$$

Vlasov-Maxwell

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f)$$

$$\frac{\partial B}{\partial t} + \nabla \times \mathbf{E} = 0$$

$$\frac{\partial E}{\partial t} - \nabla \times \mathbf{B} = -\mathbf{j}$$

Vlasov-Poisson

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f)$$

$$-\Delta \phi = \rho$$

Collisionless Vlasov-Poisson

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{q}{m}\mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$-\Delta \phi = \rho$$

Collisionless Vlasov-Poisson

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{q}{m}\mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$-\Delta\phi = \sigma + q \int_{\Omega} f \, d\mathbf{v}$$

Landau Damping

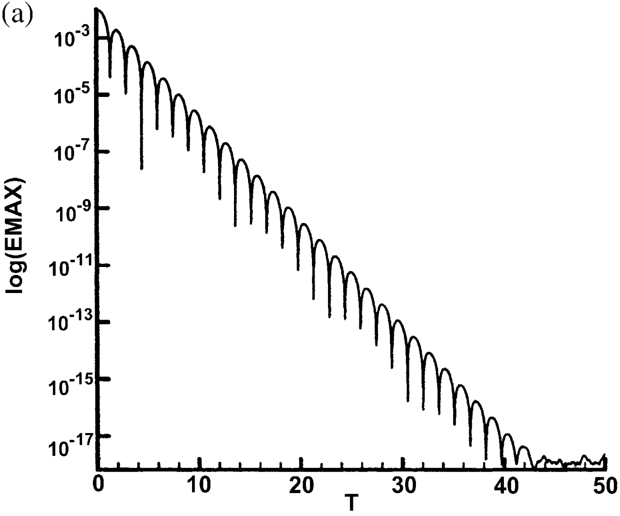
On a periodic domain $[0, 4\pi]$, perturb the density with

$$\alpha \cos(kx)$$

We measure the decay of the induced electric field for the case $k = 1/2$ and $\alpha = 0.01$ (Finn et al. 2023).

Continuum Simulation

(a)

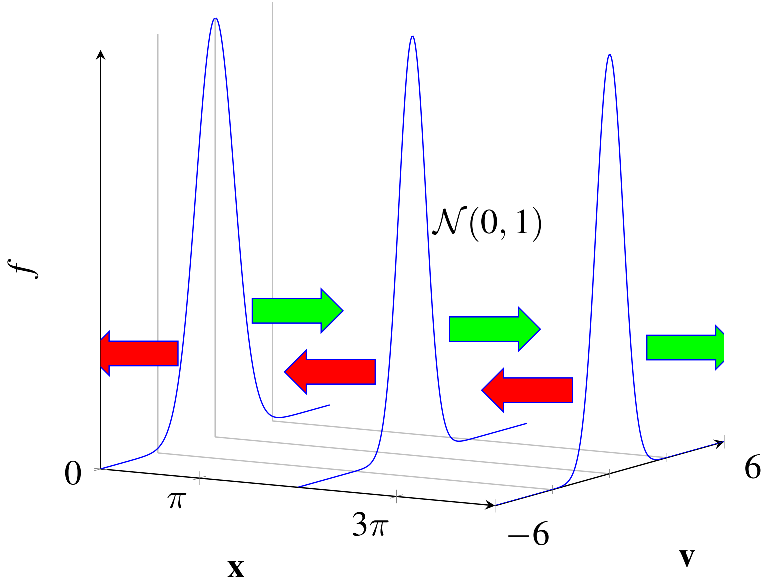


DG/FFT simulation from (Zhou, Guo, and Shu 2001)

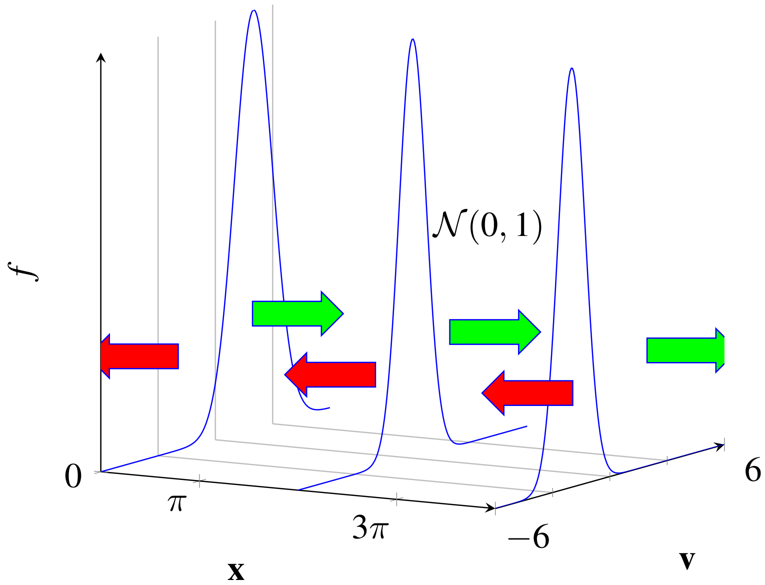
72K Particle Run

$160X \times 450V$ Grid

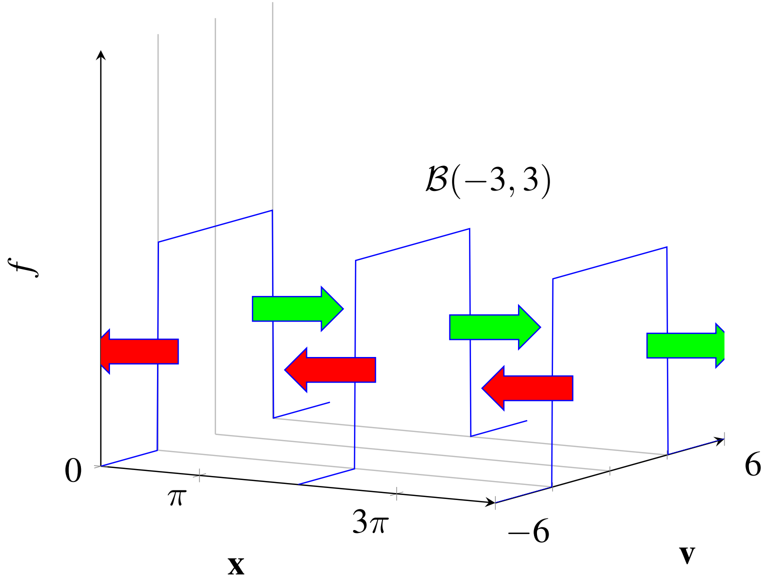
Particle Advection



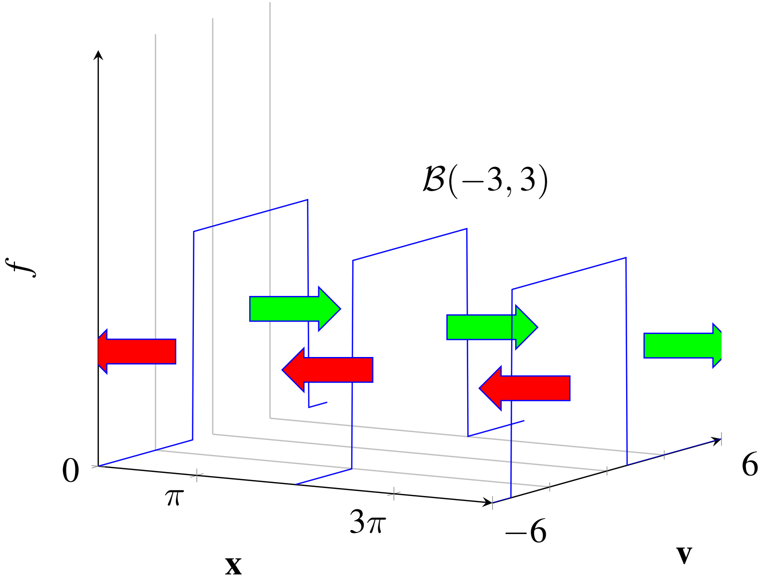
Perturbed Particle Advection



Simplified Particle Advection



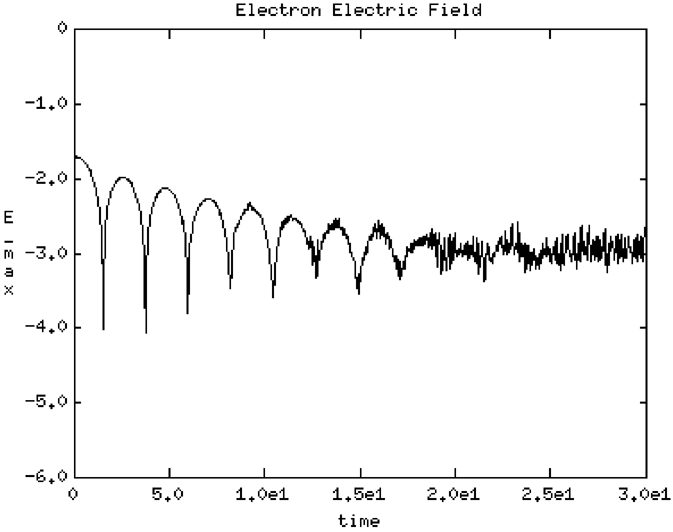
Perturbed Simplified Particle Advection



Simple Particle Exchange

Averaging

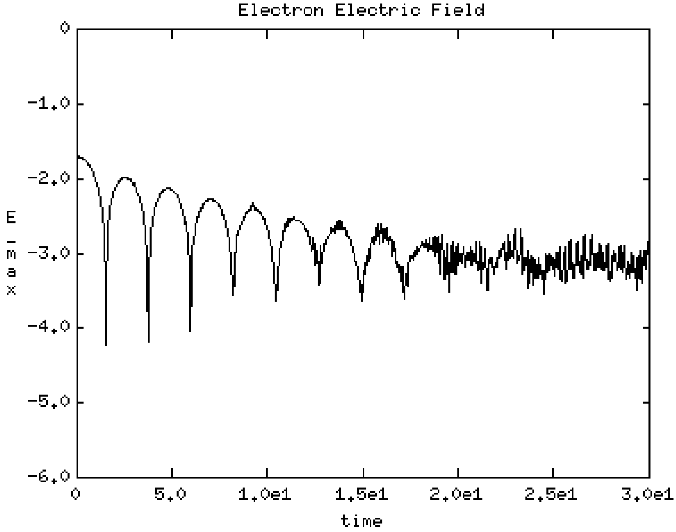
80X × 225V grid



Stencil Width: 0

Averaging

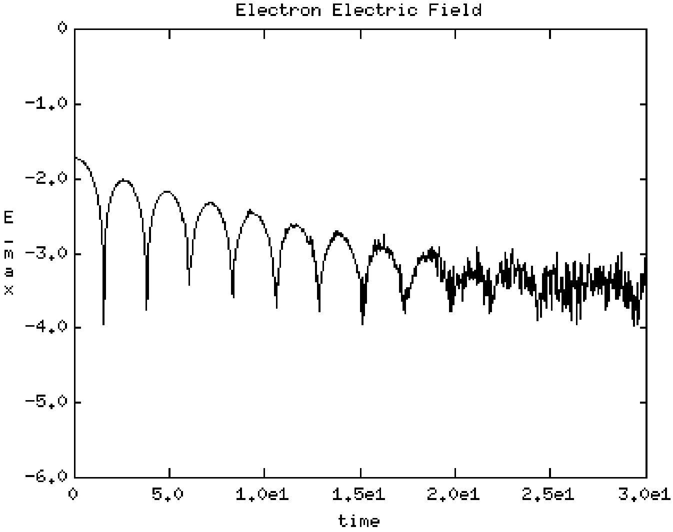
$80X \times 225V$ grid



Stencil Width: 1

Averaging

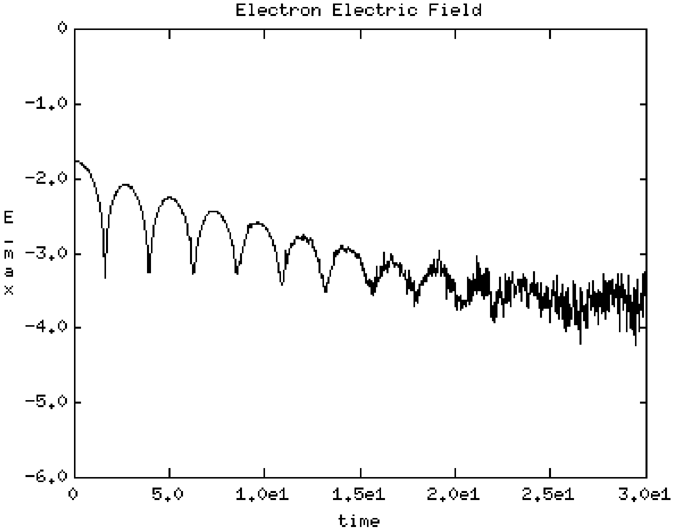
$80X \times 225V$ grid



Stencil Width: 5

Averaging

$80X \times 225V$ grid



Stencil Width: 9

Resampling

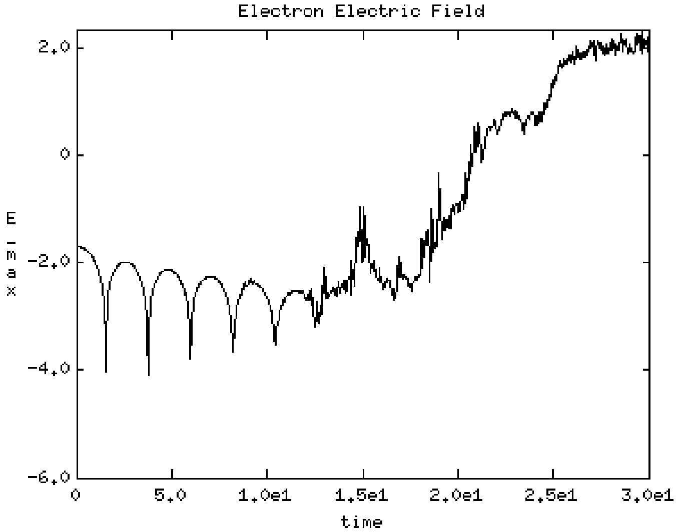
We remap particles by projecting the particle weight

$$M u_f = M_p w_p$$

to the continuum and back, using conservative projection (Pusztay, Knepley, and Adams 2022).

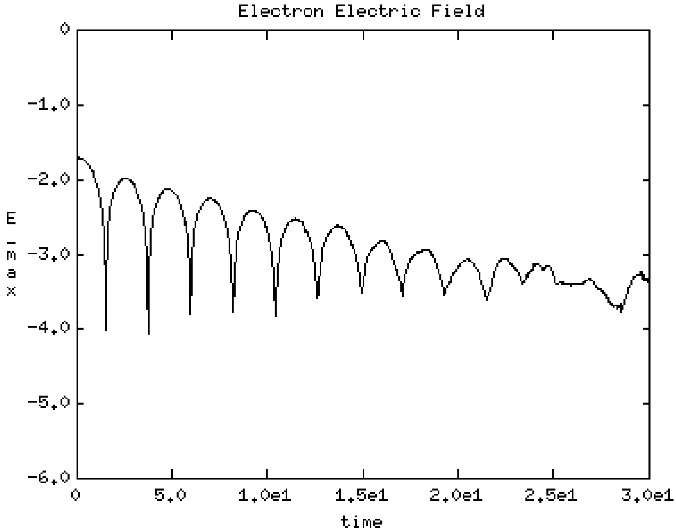
This algorithm is also used in XGC1 (Mollén et al. 2021).

Resampling



Remap grid matches particle grid

Resampling



Remap grid halves particle grid

Resampling

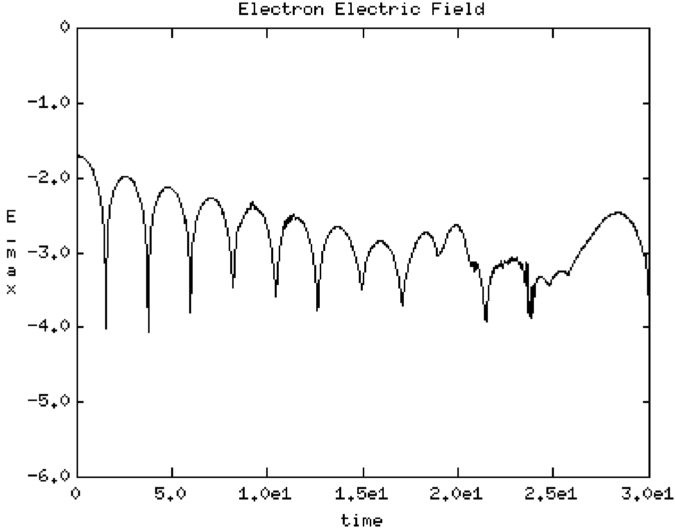
Conserving more moments results in more “noise”.

Resampling on a coarser grid reduces noise without changing the decay rate or frequency.

We can choose the remap interval and coarsening factor, but results start to worsen at some level.

Resampling

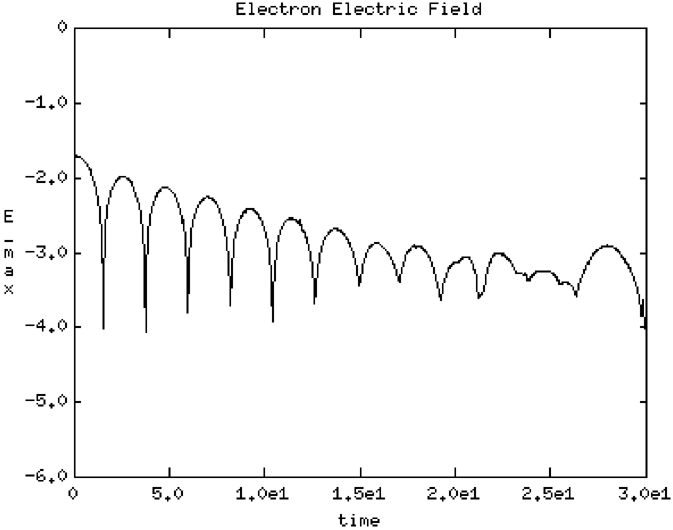
80×225 grid, 20×56 resample



Remap Interval: 12

Resampling

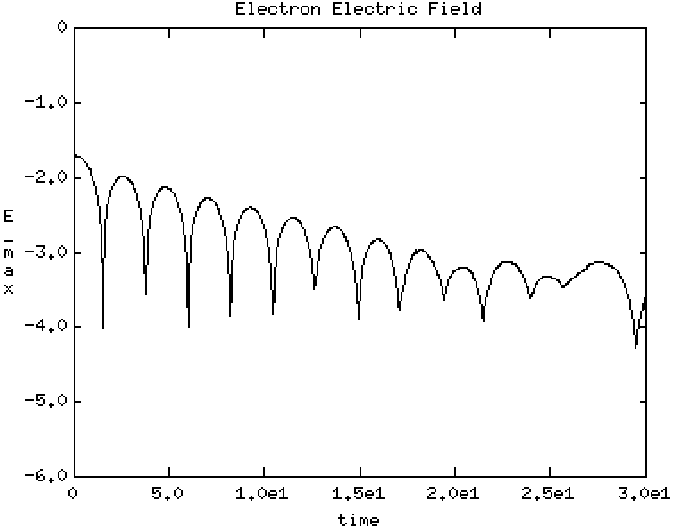
80 × 225 grid, 20 × 56 resample



Remap Interval: 6

Resampling

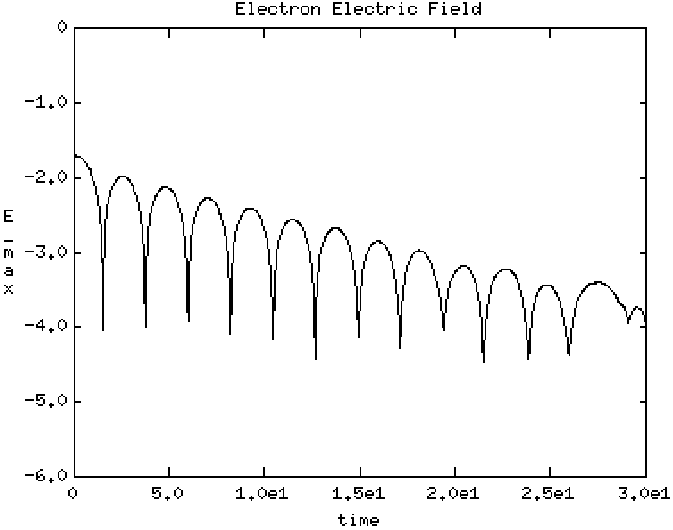
80 × 225 grid, 20 × 56 resample



Remap Interval: 3

Resampling

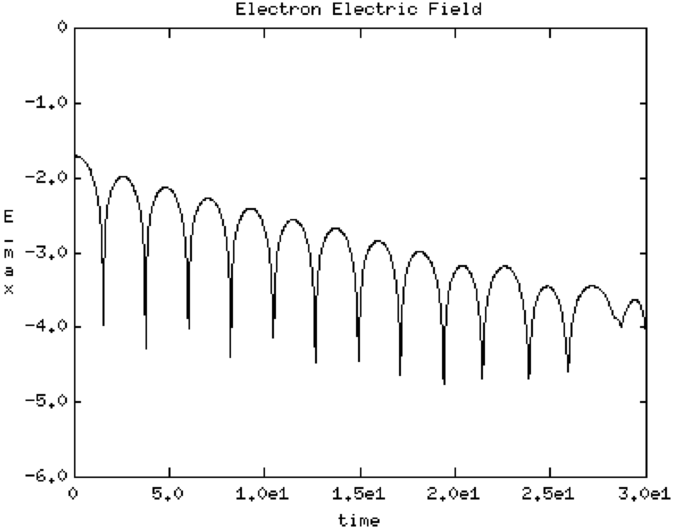
80×225 grid, 20×56 resample



Remap Interval: 1.5

Resampling

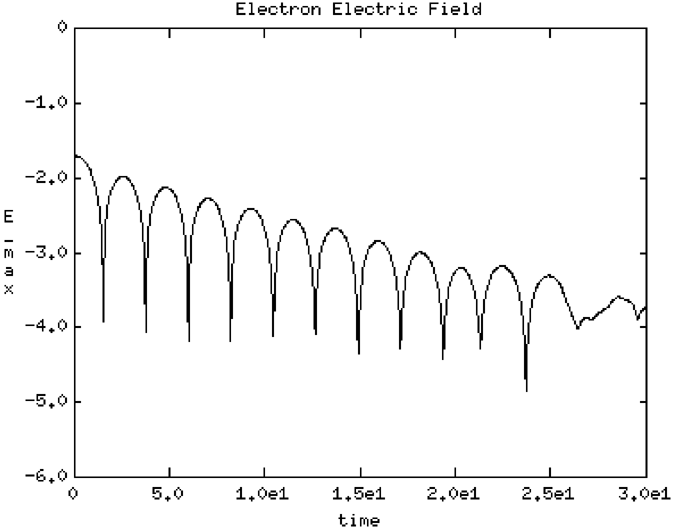
80×225 grid, 20×56 resample



Remap Interval: 0.75

Resampling

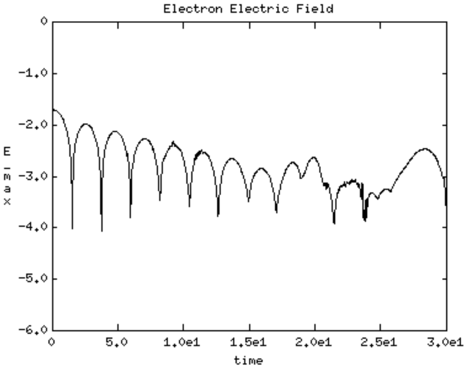
80×225 grid, 20×56 resample

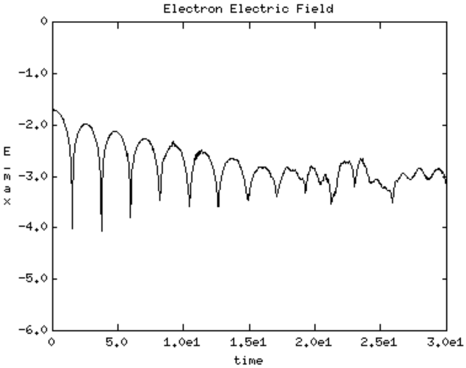


Remap Interval: 0.375

Resampling with Different Ratios

Coarsening factors 4 and 2

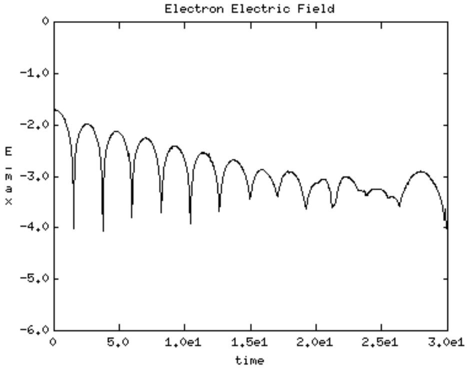


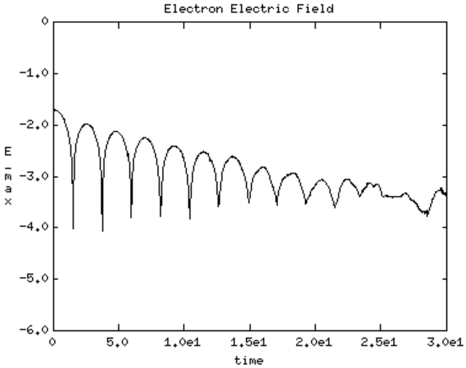


Remap Interval: 12

Resampling with Different Ratios

Coarsening factors 4 and 2

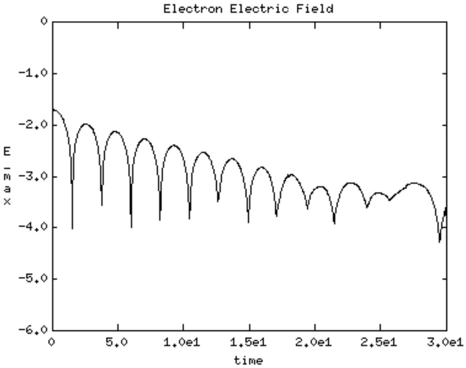


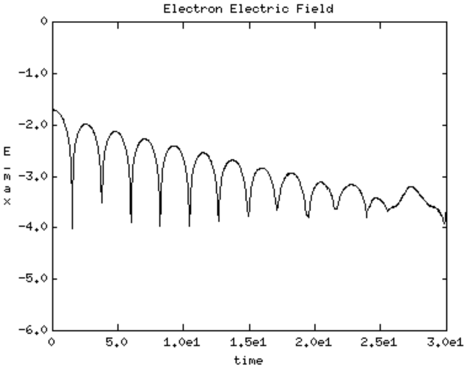


Remap Interval: 6

Resampling with Different Ratios

Coarsening factors 4 and 2

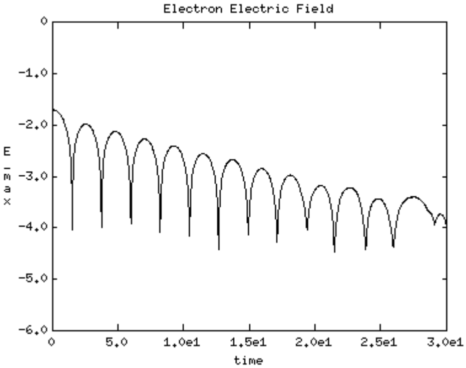


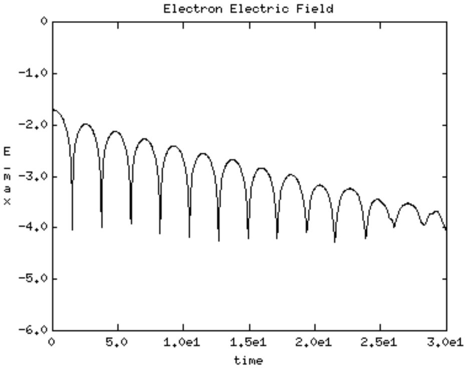


Remap Interval: 3

Resampling with Different Ratios

Coarsening factors 4 and 2

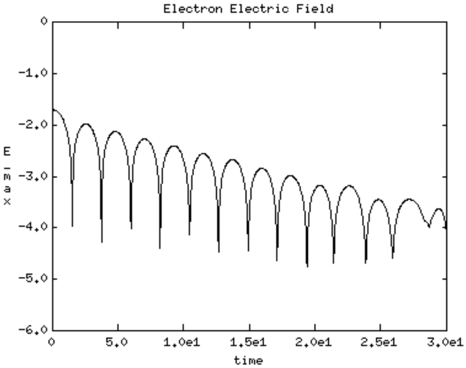


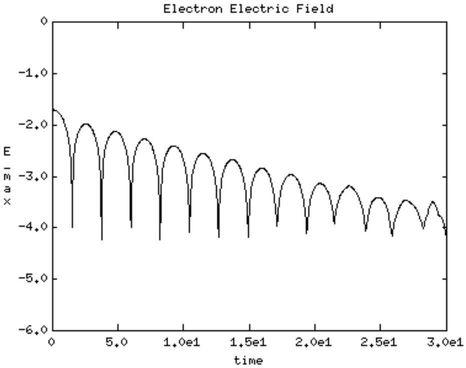


Remap Interval: 1.5

Resampling with Different Ratios

Coarsening factors 4 and 2





Remap Interval: 0.75

Path Forward

Questions:

- How do we choose the remap space?
- How do we choose the remap interval?
- How do we choose the remap resolution?

Initial experiments are in (Adams et al. 2025).

References I

Finn, Daniel S., Matthew G. Knepley, Joseph V. Pusztay, and Mark F. Adams (2023). "A Numerical Study of Landau Damping with PETSc-PIC". In: Communications in Applied Mathematics and Computational Science 18.1, pp. 135–152. DOI: 10.2140/camcos.2023.18.135. URL: https://arxiv.org/abs/2303.12620.

Zhou, Tie, Yan Guo, and Chi Wang Shu (2001). "Numerical study on Landau damping". In: Physica D: Nonlinear Phenomena 157 (4), pp. 322–333. ISSN: 01672789. DOI: 10.1016/S0167-2789(01)00289-5.

Pusztay, Joseph V., Matthew G. Knepley, and Mark F. Adams (2022). "Conservative Projection Between FEM and Particle Bases". In: SIAM Journal on Scientific Computing 44.4, pp. C310–C319. DOI: 10.1137/21M145407.

Mollén, Albert, Mark F. Adams, Matthew G. Knepley, Robert Hager, and C. S. Chang (2021). "Implementation of higher-order velocity mapping between marker particles and grid in the particle-in-cell code XGC". In: Journal of Plasma Physics 87.2, p. 905870229. DOI: 10.1017/S0022377821000441. eprint: 2012.11764.

Adams, Mark F., Daniel S. Finn, Matthew G. Knepley, and Joseph V. Pusztay (2025). "A projection method for particle resampling". In: Computer Physics Communications. Submitted. URL: https://arxiv.org/abs/2501.13681.