$$Dec(a) = \sum_{i=0}^{n-1} a_i \cdot 2^i$$

$$= \sum_{i=\lceil n/2 \rceil}^{n-1} a_i \cdot 2^i + \sum_{i=0}^{\lceil n/2 \rceil - 1} a_i \cdot 2^i$$

$$= Dec(a^1) \cdot 2^{\lceil n/2 \rceil} + Dec(a^0)$$

$\square$

$b = b_{n-1} \cdots b_0$

$Dec(b) = Dec(b^1) \cdot 2^{\lceil n/2 \rceil} + Dec(b^0)$

$b^0 = b_{\lceil n/2 \rceil - 1} \cdots b_0$

$b^1 = b_1^* \cdots b_{\lceil n/2 \rceil}$

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$$Dec(a) \cdot Dec(b) = (Dec(a^1) \cdot 2^{\lceil n/2 \rceil} + Dec(a^0)) \cdot (Dec(b^1) \cdot 2^{\lceil n/2 \rceil} + Dec(b^0))$$

$$= Dec(a^1) \cdot Dec(b^1) \cdot 2^{2\lceil n/2 \rceil} + Dec(a^1) \cdot Dec(b^0) \cdot 2^{\lceil n/2 \rceil}$$
$$+ Dec(a^0) \cdot Dec(b^1) \cdot 2^{\lceil n/2 \rceil} + Dec(a^0) \cdot Dec(b^0)$$

$$\equiv \underbrace{a \cdot b}_{\text{1 mult of n-bit numbers}} = \underbrace{a^1 \cdot b^1 \cdot 2^{2\lceil n/2 \rceil} + a^1 \cdot b^0 \cdot 2^{\lceil n/2 \rceil} + a^0 \cdot b^1 \cdot 2^{\lceil n/2 \rceil} + a^0 \cdot b^0}_{\text{4 mults of n/2-bit numbers}}$$

$$\begin{cases} T(n) \le 4(n/2) + O(n) \\ T(1) \le c \end{cases} \quad O(n^2)$$

$$= a^1 b^1 \cdot 2^{2\lceil n/2 \rceil} + 2^{\lceil n/2 \rceil} \underline{(a^1 b^0 + a^0 b^1)} + a^0 \cdot b^0$$

Key Identity

$$(a^1+a^0)(b^1+b^0) = a^1b^1 + a^1b^0 + a^0b^1 + a^0b^0$$

$$(a^1b^0 + a^0b^1) = (a^1+a^0)(b^1+b^0) - a^1b^1 - a^0b^0$$

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closest pair of points

Input: n points $P_1 \cdots P_n$; $P_i = (x_i, y_i)$

Output: P_i, P_j s.t. $d(P_i, P_j)$ is min

$$d(P_i, P_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Assumption: ① Given P_i, P_j can compute $d(P_i, P_j)$ in $O(1)$ time.

WLOG, ignore square root.

$d(P_i, P_j)$ is min $\iff$ $d(P_i, P_j)^2$ is min

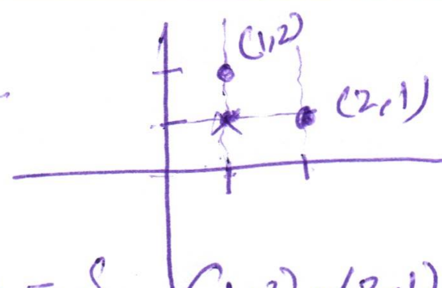
$\hookrightarrow (x_i - x_j)^2 + (y_i - y_j)^2$

② All x_i values are distinct, All y_i value " " } if not ① "rotate" all points.

② modify the algo to handle duplicate x & y values.

Notation: P is the set of all points.

$P = \{(1,2), (2,1)\}$



P_x: pts in P sorted by x values

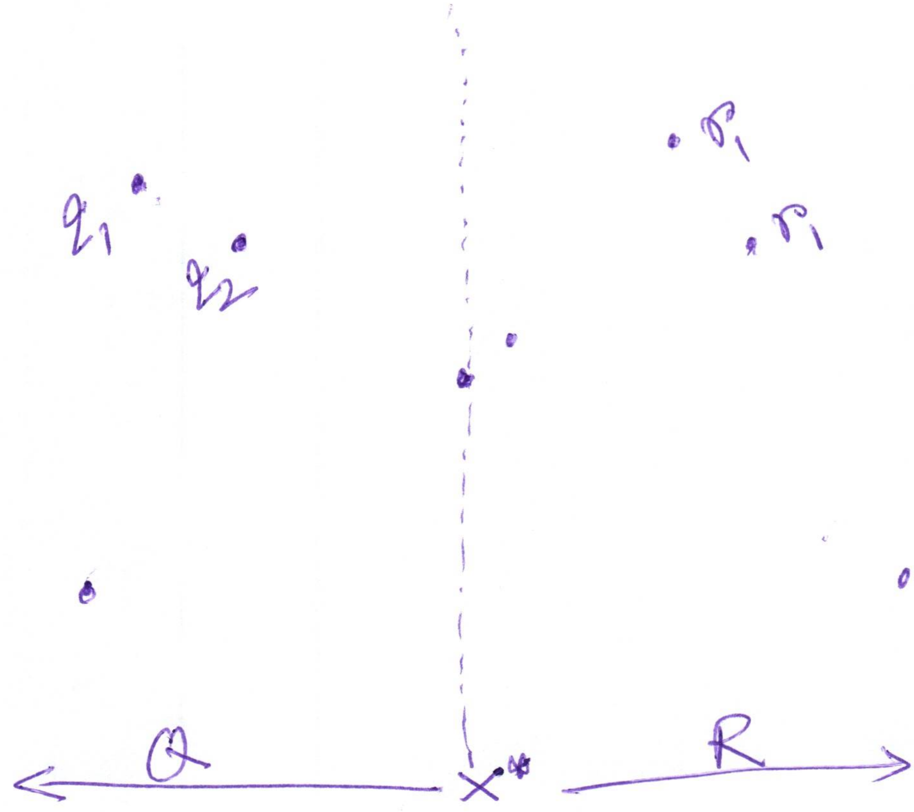
P_y: " " P " " y "

$P_x = \{(1,2), (2,1)\}$

$P_y = \{(2,1), (1,2)\}$

Towards a divide & conquer approach:

<u>n=8</u>



define $(x^*, y^*) = P_x[\lceil n/2 \rceil]$

$Q = \{(x,y) \in P \mid x \le x^*\}$

$R = \{(x,y) \in P \mid x > x^*\}$

<u>Step 2:</u>