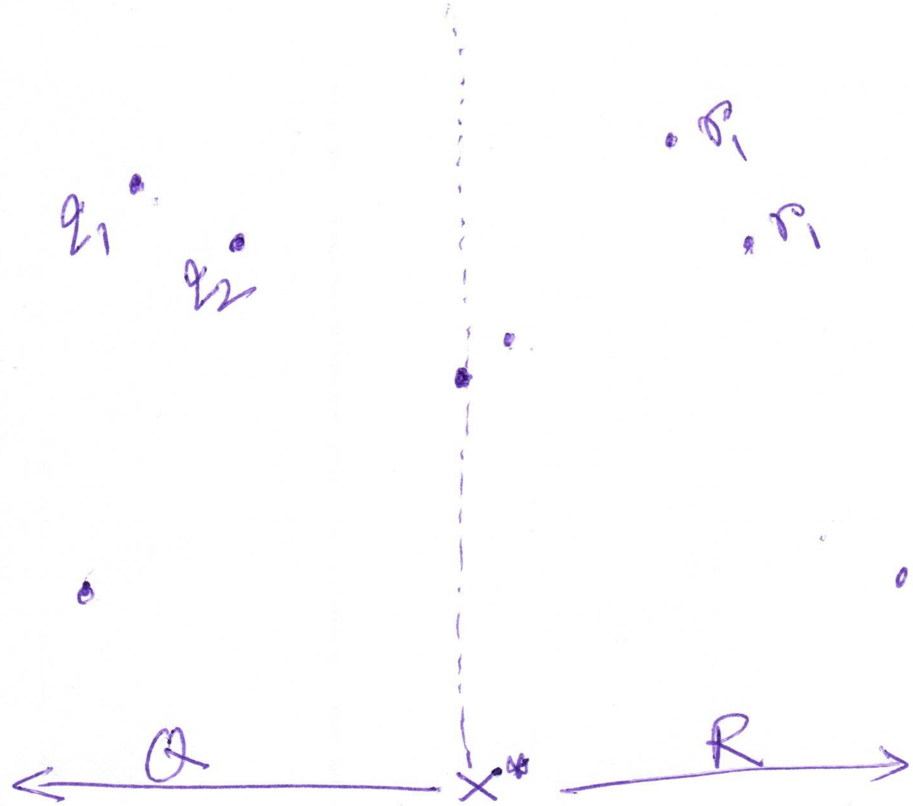
Towards a divide & conquer approach:

n = 8

Step 1



define $(x^*, y^*) = P_x[\lceil n/2 \rceil]$

$Q = \{(x,y) \in P \mid x \le x^*\}$

$R = \{(x,y) \in P \mid x > x^*\}$

Apr 14

Step 2: let (q_1, q_2) be the closest pair of points in Q
let (r_1, r_2) " " " " " " in R

Aside: Given P_x, P_y $\xrightarrow{\text{find}}$ Q_x, Q_y, R_x, R_y in $O(n)$ time.

$$\left.\begin{aligned} Q_x &= P_x[1 : \lceil n/2 \rceil] \\ R_x &= P_x[\lceil n/2 \rceil + 1 : n] \end{aligned}\right\} O(n)$$

$\to$ scan (x,y) in order of P_y.
if $x \le x^*$, add (x,y) to Q_y
else " " " R_y
$\Big\}\ O(n)$

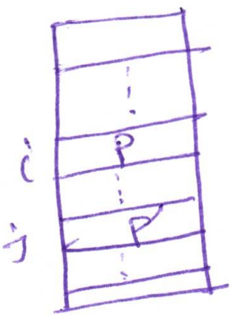
(x, y_1) (x', y_2)
(x'', y_3) ...

lemma:
(KT 5.10)

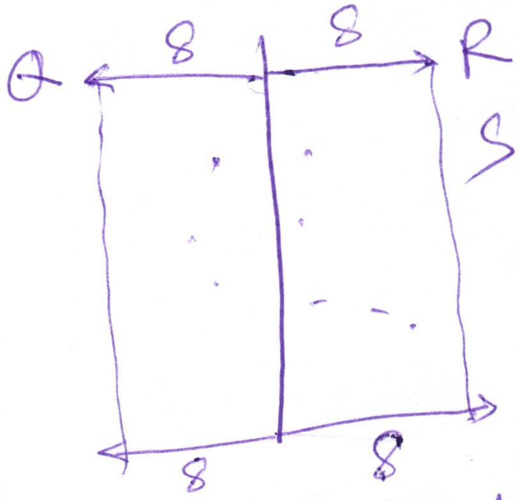
for every $p \neq p' \in S$,
s.t. $d(p, p') < \delta$

if $S_y[i] = p$
$S_y[j] = p'$

the $|i - j| \leq 15$.



S_y: pts in S sorted by y value.



$S = \{(x,y) \in P \mid |x - x^*| < \delta\}$

$x^* - \delta < x < x^* + \delta$

Note: "15" can be redou to "9", "7" ...

→ $O(n)$ time algo for closest-in-Box?

$|S| = n'$

$O(n)$ for $i = 1, \ldots, n'-1$

$O(1)$ check $(S_y[i], S_y[i+1])$, $(S_y[i], S_y[i+2])$, ... $(S_y[i], S_y[i+15])$ $\min(i+15, n')$

let (p_i, p_i') be the closest pair of points

$O(n)$ let (p, p') be closest among (p_1, p_1'), (p_2, p_2') ... $(p_{n'-1}, p'_{n'-1})$

$O(1)$ if $d(p, p') < \delta$
return (p, p')
otherwise return None