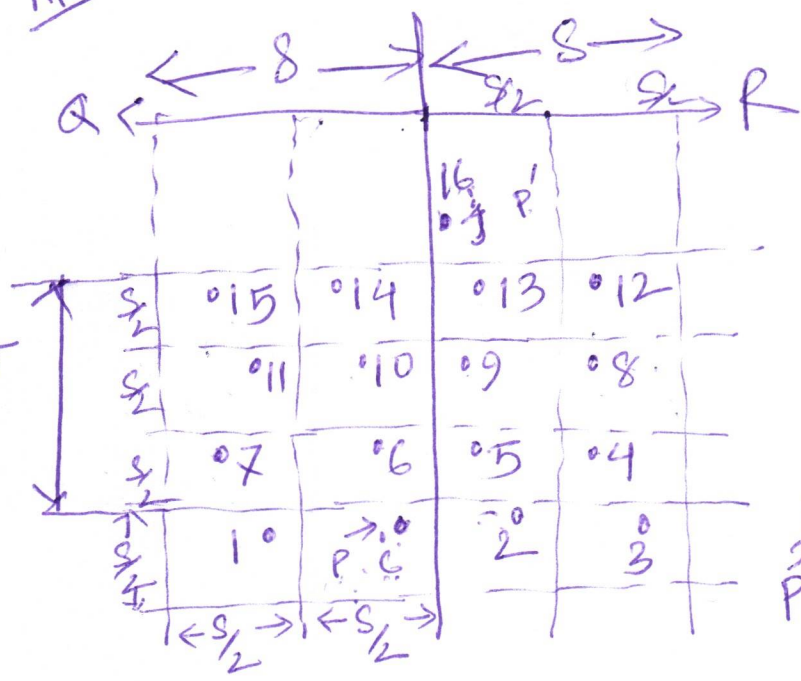
Pf. (idea)

$d(P,P') \geq 3\delta/2 > \delta$

$\Rightarrow$ Contradicts (*)

APR17



By contradiction

Assume $\exists$ $P \neq P' \in S$ s.t. (*) $d(P,P') < \delta$

$Sy[i] = P$

$Sy[j] = P'$

WLOG, $j \geq i+16$

$i, i+1, i+2, \ldots$

P $\ldots i+15, \underline{i+16}$

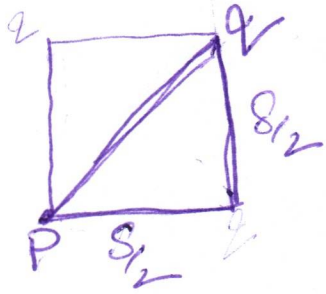
P' j

claim: Every $\delta/2 \times \delta/2$ square has at most one point P from S.

Pf. (idea): By contradiction

$\Rightarrow \exists$ a $\delta/2 \times \delta/2$ square that has two points P & Q.

Ex: P & Q are farthest apart if on diagonal.



$$d(P,Q) = \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2}$$

$$= \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{2\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$$

$\rightarrow$ Contradicts the def of $\delta \rightarrow \min(d(q_0,q_1), d(p_0,p_1))$

$\hookrightarrow$ as every $\delta/2 \times \delta/2$ box is completly inside Q or R.