APR3

MergeSort (a, n)

$$O(1) \begin{cases} \text{if } n = 1, \text{ then return } a_1 \end{cases}$$

$$O(n) \begin{cases} a_L = a_1, a_2, \ldots, a_{\lfloor n/2 \rfloor} \\ a_R = a_{\lfloor n/2 \rfloor + 1}, a_2, \ldots, a_n \end{cases}$$

$n = 5$

$\lfloor n/2 \rfloor = \lfloor 5/2 \rfloor$

$= \lfloor 2.5 \rfloor = 2$

$n - \lfloor n/2 \rfloor = 5 - 2 = 3$

$= \lceil n/2 \rceil$

$\lceil n/2 \rceil = \lceil 5/2 \rceil = \lceil 2.5 \rceil$

$= 3$

return $\underbrace{\text{MERGE}}_{O(n)} \left(\underbrace{\text{MergeSort}}_{T(\lfloor n/2 \rfloor)} (a_L, \lfloor n/2 \rfloor), \underbrace{\text{MergeSort}}_{T(n - \lfloor n/2 \rfloor)} (a_R, n - \lfloor n/2 \rfloor) \right)$

$T(n) \stackrel{\text{def.}}{=}$ max. runtime of MergeSort over all inputs of size n.

$$T(n) \leq O(1) + O(n) + T(\lfloor n/2 \rfloor) + T(n - \lfloor n/2 \rfloor) + O(n)$$

if $n = 1$, $T(n) \leq O(1)$

if $n > 1$, $T(n) \leq O(n) + T(\lfloor n/2 \rfloor) + T(n - \lfloor n/2 \rfloor)$

$$T(n) \leq \begin{cases} O(1), & \text{if } n = 1 \\ O(n) + T(\lfloor n/2 \rfloor) + T(n - \lfloor n/2 \rfloor), & \text{o/w} \end{cases}$$

By def. of Big Oh, $\exists$ constants c_1, c_2

$$T(n) \leq \begin{cases} c_1, & \text{if } n = 1 \\ c_2 n + T(\lfloor n/2 \rfloor) + T(\underbrace{n - \lfloor n/2 \rfloor}_{\lceil n/2 \rceil}), & \text{o/w} \end{cases}$$

pick $c = \max(c_1, c_2)$

$$T(n) \le \begin{cases} c, & \text{if } n=1 \\ cn + T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil), & \text{o/w} \end{cases}$$

Rule of thumb: for asymptotics of $T(n)$ enough to show: $T(\lfloor x \rfloor) \Rightarrow T(x)$, $T(\lceil x \rceil) \Rightarrow T(x)$

$$T(n) \le \begin{cases} c, & \text{if } n=1 \\ cn + 2T(n/2), & \text{o/w} \end{cases}$$

Lemma: $T(n) \le cn\log_2 n + cn$ $\quad (\le O(n\log n))$

$\Rightarrow$ MergeSort runs in $O(n\log n)$ time.