Apr 7

Collaborative filtering (Netflix)

~~Use~~ Each user ≡ a ranking of movies/shows on Netflix.

Hypothesis: User A is close to User B if their rankings are close.

User 1	User 2	User 3
① Mx	③ - 1	① - 1
② My	② - 2	③ - 2
③ Mz	① - 3	② - 3

(2,3)

Assume: Each user ranks all movies/shows.

Input: A ranking $a_1 \cdots a_i a_j \cdots a_n$ → (a permutation of $1 \cdots n \stackrel{\text{def}}{=} [n]$)

(Implicit assumption is that $1, 2, \cdots, n$ is the "true" ranking).

Output: ~~#~~ number of inversions.

Def: A pair (i, j) is an inversion if (I) $i < j$ AND (II) $a_i > a_j$

Ex 1: User 2 - How many inversions? {(1,2), (1,3), (2,3)}

↳ every pair is an inversion: $\binom{3}{2} = 3$ inversions.

User 3: (2,3) is an inversion

∴ 1 inversion.

Ex 2: $a = (1, 2, \ldots, n)$ #inversion $= 0$

↳ (increasing) sorted input: 0 inversion.

Ex 3: $a = (n, n-1, \ldots, 1)$

$$\#\text{inversion} = \#\text{pairs} = \binom{n}{2} = \frac{n(n-1)}{2}$$

$$0 \leq \#\text{inversions} \leq \binom{n}{2}$$