Feb8

Gale-Shapley Algorithm

(0) Initially, all n men and n women are free

(1) In a loop: (in textbook: men propose) → a free woman proposes to a man (things happen)

(2) You have n matched pairs

n=2; M={BP, BBT}, W={JA, AJ}

L_{JA}: BP > BBT ; L_{BP}: AJ > JA

L_{AJ}: BBT > BP ; L_{BBT}: JA > AJ

Initial State: all n men + n women are free

F: Free ; E: Engaged

AJ	JA	BP	BBT
F	F	F	F

① Let w be a free woman

(Q1) which man m should w propose to?

A1: the man m on top of L_w

(Q2) what should m do? (w→m) proposed.

Accept? ×

Reject? ×

(m, w) get engaged.

(Q1) who should JA propose to?

A1: BP

(JA → BP) proposed

(Q2) what should BP do?

Accept? × (BP, JA) get engaged

Reject? ×

Feb10

In general: Either all men/women are engaged or some are free.

① All n men & women are engaged.

AJ	JA	BP	BBT
F	E	E	F

Feb 10

(Q1) what should algo. do?

A1: Algo. terminates & output n engaged pairs.

(2) ∃ a free woman w

Q1: who should w propose to?

A1: The best man m in L_w that w has not proposed to yet. ~~m → w~~ (w → m) proposed.

Q2: what should m do?

Case 1: m is free.

(m, w) get engaged.

Case 2: m is engaged to w'. ($w \neq w'$).

Case 2.1: $w' > w$ in L_m

⇒ nothing changes.

Case 2.2: $w > w'$ in L_m.

(m, w) get engaged.

w' is free.

Algo. continues execution

F: Free; E: Engaged

AJ	JA	BP	BBT
F	E	E	F

(Q1) who should AJ propose to?

(A1) BBT

(AJ → BBT) proposed.

(Q2) what should BBT do?

(BBT, AJ) get engaged

AJ	JA	BP	BBT
E	E	E	E

⇒ Algo. terminates & output 2 engaged pairs.

⇒ {(BP, JA), (BBT, AJ)}