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THEOREM: For every input (M, W, 2n pref. lists), the GS algo. outputs stable matching. | $|M| = |W| = n$

Corollary: Every input to a stable matching problem has a stable matching. (Run GS)

Pf: - follows from the theorem.

Pf. of Theorem

Let S be the output of GS algo. on an arbitrary input.

Want to argue: S is a stable matching.

Lemma 1: For every input, GS terminates in $\leq n^2$ iteration.

Lemma 2: S is a perfect matching.

Lemma 3: S has no instability.

Lemmas 1+2+3 $\Rightarrow$ THEOREM

Pf. (idea) of lemma 1: In each iteration, a new proposal is made from $w \to m$, $w \in W$, $m \in M$.

$\#$ iterations $= \#$ proposals $\leq \#$ pairs $(m, w) = |M \times W|$

$= |M| \times |W|$

$= n \times n = n^2$

$\#$ iterations $\leq n^2$

$\square$

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Obs 0 : S is a matching.

Obs 1 : Once a man gets engaged, he keeps getting engaged to better women.

Obs 2 : If w proposes to m after m', m' > m in L_w

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lemma 4 : If at the end of an iteration, w is free, she has NOT proposed to all men.

Pf. (idea) of lemma 2: proof by contradiction + use (Obs 0 + algo. def. + lemma 1 + lemma 4).

Pf. (details) : Assume that S is not a perfect matching.

$\Longrightarrow$ $\exists$ a free woman.

by (obs 0 + algo. def) $\Longrightarrow$ by lemma 4 $\exists$ a man n that w has not proposed to (*)

By lemma 1, GS terminates $\Longrightarrow$ All free women proposed to all men

$\Longrightarrow$ Contradicts (*)

$\square$