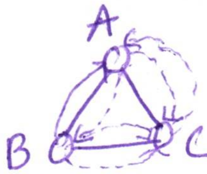
Feb 22

Graphs

$G = (V, E)$

set of vertices (V); set of edges (E)

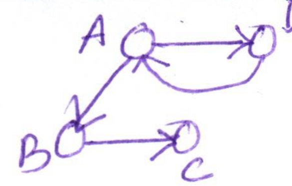


$V = \{A, B, C\}$

$E = \{(A,B), (B,C), (C,A), (B,A), (C,B), (A,C)\}$

$n = |V| = 3$

$m = |E| = 3\ (6)$



$V = \{A, B, C, D\}$

$E = \{(A,B), (B,C), (A,D), (D,A)\}$

$n = 4$

$m = 4$

Default: $|V| = n$; $|E| = m$

Def: G is undirected

$\Longleftrightarrow \forall\, u \neq w\ (u,w) \in E \Leftrightarrow (w,u) \in E$

iff

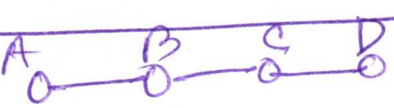
(Q1) Airline map (?) (Q2) Wiki articles (directed)

Default: G is undirected.

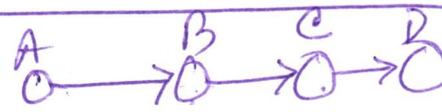
claim: Every undirected graph is also a directed graph.

(Pf: idea): Replace

path:



	length
D, C, B, A ✓	3
A, B, C, D ✓	3
A, B, C, B ✓ (1, 2, 3)	3
A, C, D ✗	—



D, C, B, A ✗

A, B, C, D ✓

A, B, C, B ✗

A, C, D ✗

Def: A path in $G = (V, E)$ is a sequence of vertices $u_1, \dots, u_k$ {$u_1 - u_k$ path} s.t. $\forall i \in [k-1] = \{1, \dots, k-1\}$, $(u_i, u_{i+1}) \in E$.

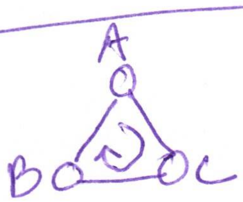
Notes: (i) u_i need not be distinct.
(ii) holds for directed graph.

Def: A simple path is a path with no repeated vertices.

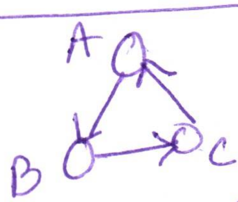
Ex: A simple path of n nodes has $\le n-1$ edges.
↳ has length $\le n-1$.

Def: The length of a path is the number of edges in it.

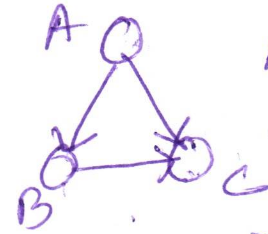
Cycles:



A, B, C, A ✓
A, C, B, A ✓



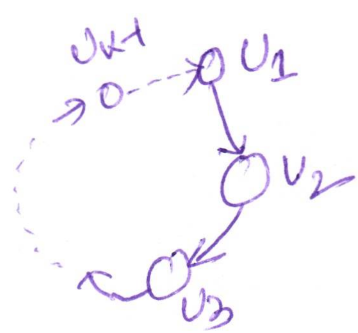
A, B, C, A ✓
A, C, B, A ×



A, B, C, A ×
A, C, B, A ×

DAG: Directed acyclic graph

Def: A cycle is a sequence of vertices $u_1, \dots, u_k = u_1$ {$(u_1, \dots, u_{k-1}$ are distinct)} s.t. $\forall i \in [k-1] = \{1, \dots, k-1\}$
$(u_i, u_{i+1}) \in E$



(i) Directed: $k = 3$

(ii) undirected: $k = 4$