Feb 24

Proposition: Let T be a BFS tree in $G = (V, E)$.

If $(u, w) \in E$ s.t. $u \in L_i$, $w \in L_j$

$\Rightarrow |i - j| \le 1 \Leftrightarrow i \in \{j-1, j, j+1\}$

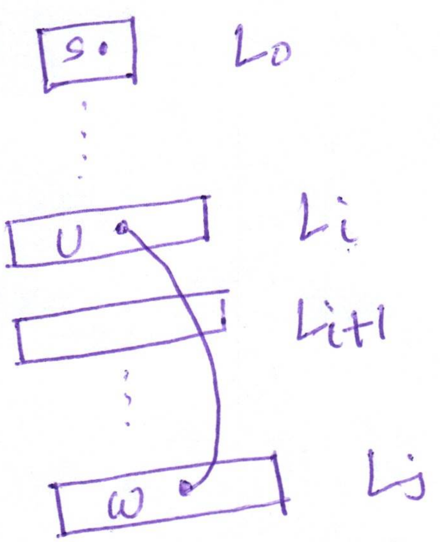
(Pf. idea): WLOG (without loss of generality) assume $i \le j$ [if $i > j$, change the roles of i & j]

By contradiction

For contradiction, assume $|i-j| > 1$

$\Rightarrow j > i+1$

$\Rightarrow j \geq i+2$



Consider a time when BFS is creating L_{i+1}

$\rightarrow u \in L_i$, $w \notin L_0, \ldots, L_i$

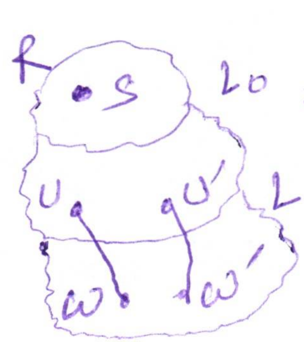
$\rightarrow (u, w) \in E$

$\Rightarrow$ BFS will add w to L_{i+1}

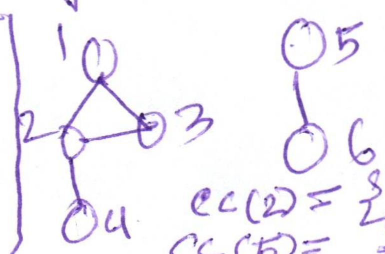
$\Rightarrow$ Contradicts $w \in L_j$ with $j \geq i+2$

Explore (s)

0. $R \leftarrow \{s\}$
1. while $\exists (u, w)$ s.t. $u \in R, w \notin R$
   add w to R
2. Output $R^* \leftarrow R$



Def: Set of all vertices connected to s is called the connected component of $s \rightarrow cc(s)$



$cc(2) = \{1, 2, 3, 4\}$

$cc(5) = \{5, 6\}$

THEOREM: for all G, all start vertices,

set → $CC(s) = R^*$ ← set

Corollary: BFS is correct.

↑ BFS is a special case of Explore.

General idea: $\underset{\text{set}}{A} = \underset{\text{set}}{B} \iff A \subseteq B$ AND $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s)$ [Everything that is output by Explore is correct (i.e. in $CC(s)$)]

Lemma 2: $CC(s) \subseteq R^*$ → [Everything that is supposed to be output (i.e. $CC(s)$) is output by Explore].