THEOREM: for all G, all start vertices s,

$CC(s) = R^*$ (set)

Corollary: BFS is correct.

↑ BFS is a special case of Explore.

General idea:

$A = B \iff A \subseteq B$ AND $B \subseteq A$ (set, set, subset)

Lemma 1: $R^* \subseteq CC(s)$ [Everything that is output by Explore is correct (i.e. in $CC(s)$)

Lemma 2: $CC(s) \subseteq R^*$ → [Everything that is supposed to be output (i.e. $CC(s)$) is output by Explore].

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Lemmas 1+2 ⇒ Thm.

Pf. of lemma 1: Ex. (by induction).

Pf (idea) of lemma 2: By contradiction.

Note: since R^* is output by Explore ⇒ Explore has terminated. (*)

Assume $CC(s) \not\subseteq R^*$

⇒ $\exists\ w \in CC(s)$ (in) but $w \notin R^*$

⇒ $\exists$ s-w path P in G but $w \notin R^*$

aside: $\{a,b,c\}^A \not\subseteq \{a,b\}^B$

Since p starts inside R^* (as $s \in R^*$)

but p ends up outside of R^* (as $w \notin R^*$)

$\Rightarrow$ p has to "cross" the boundary of R^* at some point.

$\Rightarrow$ $\exists (x, y) \in E$ s.t. $x \in R^*$, $y \notin R^*$

$\Rightarrow$ y should have been added to R' by Explore

$\Rightarrow$ Explore should not have terminated.

$\Rightarrow$ Contradicts (*) $\square$