Feb 6

Stable Matching

$n = 2$

$M = \{BP, BBT\}$

$W = \{JA, AJ\}$

Preference list:

L_{BP}: $AJ > JA$

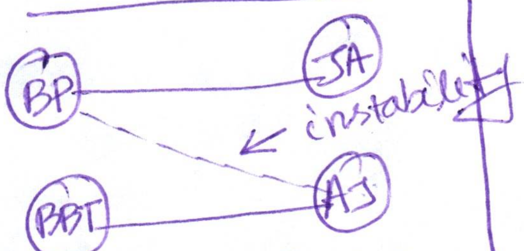
L_{BBT}: $AJ > JA$

L_{JA}: $BP > BBT$

L_{AJ}: $BP > BBT$

Def (Pref. list): $\forall m \in M$, L_m: a total ranking of all n women

$\forall w \in W$, L_w: a total ranking of all n men.



Stable? NO

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Def (Stable Matching): A perfect matching with no instability.

General n:

(Q1) How many pref. lists if n men and n women?

A1: $2n$

(Q2) How many elements in total across all pref. lists?

[How many memory slots do you need to store all pref. lists?]

A2: # pref. lists $\times$ |pref. list|

$= 2n \times n = 2n^2 = \Theta(n^2)$

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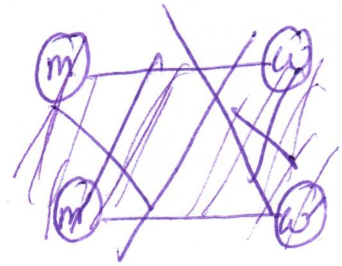
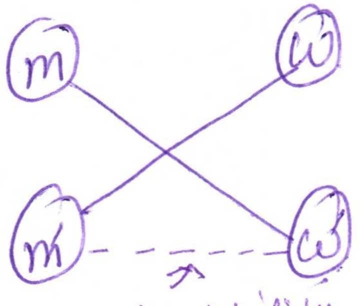
Def (Instability): Given $2n$ pref. lists and a perfect matching $S \subseteq M \times W$, an unmatched ~~A~~ pair (m', w') is an instability IF

(i) $m' > m$ in $L_{w'}$ AND
(ii) $w' > w$ in $L_{m'}$





$S = \{(m, w'), (m', w)\}$

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Gale-Shapley Algorithm

(0) Initially, all n men and n women are free

① In a loop: In textbook: men propose
a free woman proposes to a man
(things happen)

② you have n matched pairs

Initial State: all n men + n women are free

① let w be a free woman
(Q1) which man m should w propose to?
A1: the man m on top of L_w
(Q2) what should m do?
Accept?
Reject?

$n=2$; $M=\{BP, BBT\}$, $W=\{JA, AJ\}$
L_{JA}: $BP > BBT$; L_{BP}: $AJ > JA$
L_{AJ}: $BBT > BP$; L_{BBT}: $JA > AJ$

F: Free

AJ	JA	BP	BBT
F	F	F	F

(Q1) who should JA propose to?
A1: BP
(JA → BP) proposed
(Q2) what should BP do
Accept?
Reject?