Mar 10

Input: Directed graph $G = (V, E)$

$s \in V$

"lengths" $l_e \geq 0$ ← integers

Output: $\forall t \in V$ output a shortest s-t path ← w.r.t. the length of path.

$$l(P) = \sum_{e \in P} l_e$$

Simpler version: $\forall t \in V$ ~~d~~ output $d(t)$ ← length of a shortest s-t path.

Special case: $l_e = 1$ ~~$\forall$~~ $e \in E$

$\equiv$ HW3 Q3 ~~at~~

Q. $l_e = L$ ~~e~~ $\forall e \in E$ $(L \geq 1)$

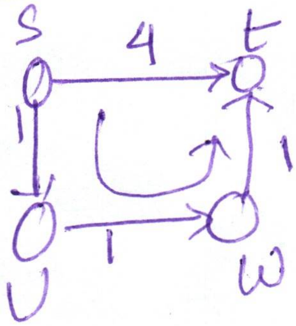
A. ignore L and assume $l_e = 1$.

→ replace L by 1.

General case: $l_e \geq 0$ $\forall e \in E$ (l_e can be different)

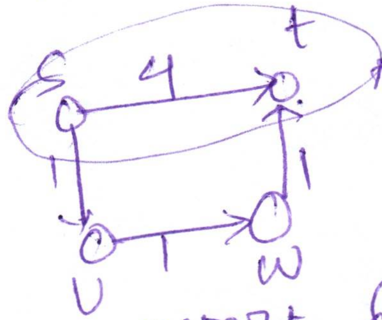
Algo idea: Reduce the general case to $l_e = 1$ and run HW3 Q3 algo.

idea 1: ignore l_e, assume $l_e = 1$.



$d(t)=3$

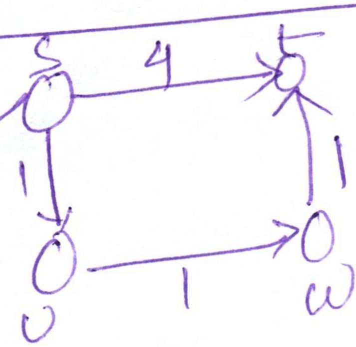
ignore l_e



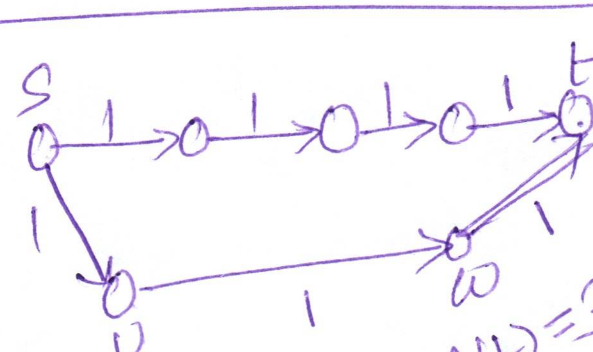
shortest path according to HW3 Q3 algo

report $d(t)=4$

idea 2

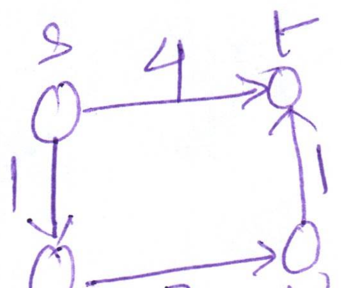


G

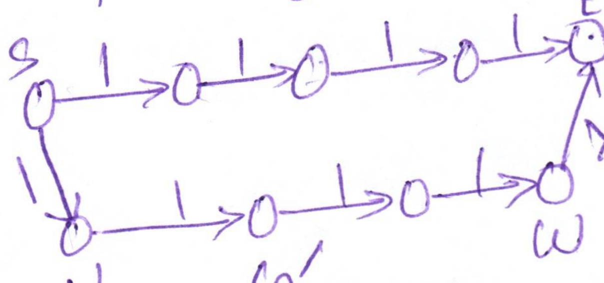


G′ $d(t)=3$

HW3 Q3 algo will pick s–u–w–t



G



G′

$d(t)=4$

Algo idea: Replace each edge $e \in E$ by a path of length l_e. (Note: New paths do not share any edges/nodes)

$\Rightarrow$ Run HW3 Q3 algo.

Claim: a shortest s-t path in G
$\Leftrightarrow$ equivalent shortest s-t path in G'

Correctness: claim + correctness of HW3 Q3 algo

Runtime: $O(|V'| + |E'|)$ $\quad |V'| = n'$

$O(m' + n')$ $\quad |E'| = m'$