OUTPUT : $E' \subseteq E$ s.t. T is a subgraph

(i) $T = (V, E')$ is connected

(ii) $c(T) = \sum_{e \in T} c_e$ is ~~the~~ minimized.

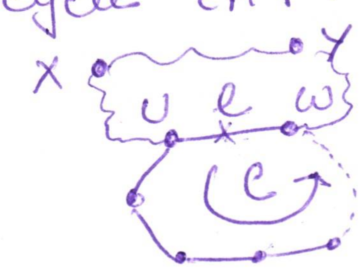
Mar 29

PROP : let $c_e > 0$ $\forall e \in E$, then any optimal solution $T = (V, E')$ is a tree.

Pf. (idea): By contradiction.

Assume $T = (V, E')$ is an optimal solution, but T is not a tree. $\rightarrow$ (*)

Since T is connected $\Rightarrow \exists$ a cycle in T.



let e be any edge in C.

delete e from T.

~~$C(T$~~) $\quad T' = (V, E' \setminus \{e\})$

claim 1 : $c(T') < c(T)$, $c(T') = c(T) - c_e < c(T)$ as $c_e > 0$

claim 2 : T' is still connected.

Case 1: $\exists$ an x-y path that doesn't use $e = (u, w)$. It

Case 2: All X–Y paths that use $e = (u, w)$

$\Rightarrow$ use the rest of C to connect u & w.

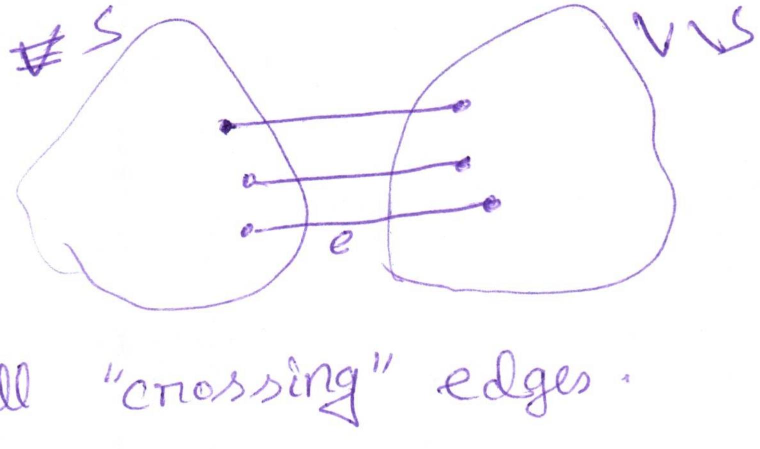
$\Rightarrow$ X, Y are still connected.

claim 1 + claim 2 $\Rightarrow$ T' is a better solution than T. $\Rightarrow$ Contradicts T is optimal. $\square$
(*)

Cut Property lemma

Assume all c_e's are distinct.

For all cuts $(S, V \setminus S)$, $S \neq \emptyset$, $V \setminus S \neq \emptyset$
$\hookrightarrow S \neq V$



Consider all "crossing" edges.
Let e be the cheapest crossing edge.
$\Rightarrow$ e is in ALL MSTs.