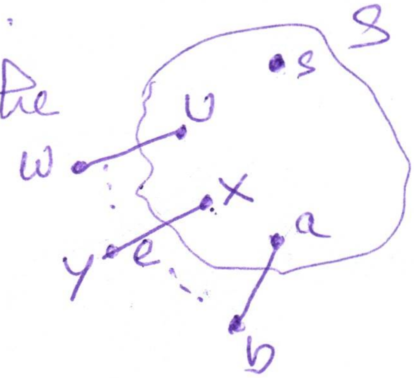
Mar 31 Assume cut Property lemma is correct + all c_e's are distinct.

THM1: Prim's algo is correct.

Consider a run of Prim's algo when it is about to add edge e to T.

Goal: will show that e is the cheapest crossing edge across some cut.



Apply cut property lemma to the cut $(S, V\setminus S)$ where S is in prim's ~~alg~~ algo.

claim 1: $S \neq \emptyset$ (as $x \in S$)

claim 2: $V\setminus S \neq \emptyset \Rightarrow S \neq V$ (as $y \notin S$)

claim 3: e is the cheapest crossing edge. (from prim's algo defn)

$\Rightarrow$ every edge added to T by Prim's algo is a correct/safe edge. (as e is in ALL MSTs)

$\Rightarrow$ Need to show that T is connected.

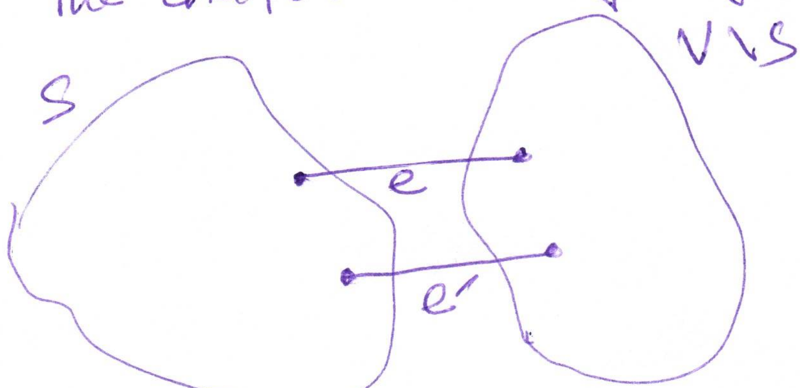
claim 4: At the end of each iteration, (S, E') is connected.

$\Rightarrow$ At the end $T = (V, E')$ is connected

claims 1+2+3+4 + cut property lemma $\Rightarrow$ THM 1 $\square$

Pf. (idea): [Cut Property Lemma]: By contradiction.

Assume $\exists$ a cut $(S, V \setminus S)$ and an MST $\rightarrow (*)$
$T = (V, E')$ s.t. the cheapest crossing edge e is not in T.



since T is connected $\Rightarrow \exists$ a crossing edge $e' \in E'$.

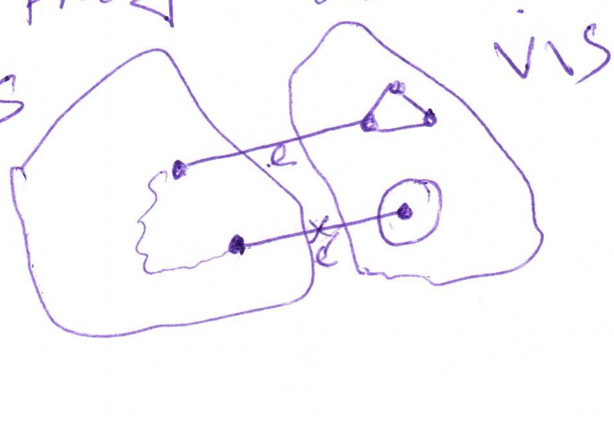
define $T' = (V, (E' \setminus \{e'\}) \cup \{e\})$

$c(T') = c(T) - c_{e'} + c_e \rightarrow$ [$c_{e'} > c_e$ as e is the cheapest crossing edge + all c_e's are distinct]

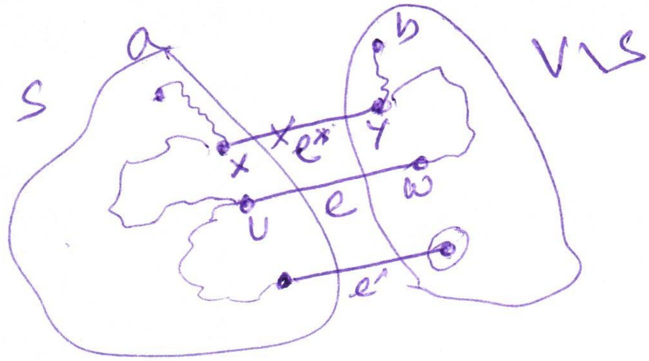
$< c(T)$

$\Rightarrow$ Contradicts $(*)$. $\square$

* Q: ~~pf~~ what's wrong with the ~~abo~~ "proof" above?



fix: choose e' carefully.



Since T is connected $\Rightarrow \exists$ u-w path in T'

as $u \in S$ but $w \notin S$. $\Rightarrow \exists$ a crossing edge $e^* = (x, y)$.

Define $T' = (V, (E' \setminus \{e^*\}) \cup \{e\})$

<u>claim 1:</u> $c(T') < c(T)$

$$c(T') = c(T) - c_{e^*} + c_e$$
$$< c(T) \quad \text{as } c_{e^*} > c_e$$

+ all c_e's are distinct

<u>claim 2:</u> T' is connected.

<u>Case 1:</u> a-b path doesn't use e^*.

<u>Case 2:</u> a-b path uses e^*.

$\Rightarrow$ take the longer route.

$\square$