Mar 8

THM 2: $|S^*| = |O|$

Notation: $S^* = \{i_1, i_2, \dots, i_K\}$ $f(i_1) \le f(i_2) \le \dots \le f(i_K)$

$O = \{j_1, j_2, \dots, j_m\}$ $f(j_1) \le f(j_2) \le \dots \le f(j_m)$

THM 2' $K = m$

claim 1: $K \le m$ (as O is an optimal solution)

Lemma 1: (Greedy stays ahead) $\forall\, l$ $1 \le l \le K$

$f(i_l) \le f(j_l)$.

[Assume lemma 1 is true]

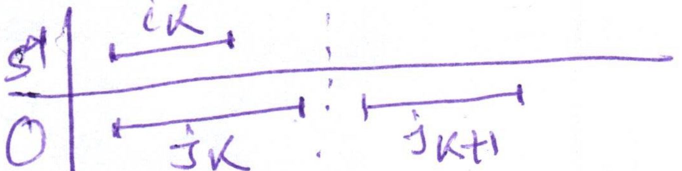
Pf. (idea) of THM 2': By Contradiction.

Assume $K \ne m \Rightarrow m \geq K+1 \Rightarrow j_{K+1} \in O$.

By lemma 1, $f(i_K) \le f(j_K)$

Consider a situation right after greedy algo adds i_K to S.

[Note: i_K is the last interval added to S] (*)



[j_{K+1} does not conflict with $i_K, i_{K-1}, \dots, i_1$]

$\Rightarrow j_{K+1} \in R$

$\Rightarrow R \ne \emptyset$

$\hookrightarrow$ contradict (*)

$S = \{i_1, i_2, \dots, i_K\}$

$R = \{\dots\dots j_{K+1}\}$

Pf. (idea) of lemma 1: by induction on l.

Base case: $f(i_1) \le f(j_1)$ ← By algo. defn & $i_1 = 1$ & $f(i)$ is the smallest finish time.

IH: Assume for some $r \geq 1$

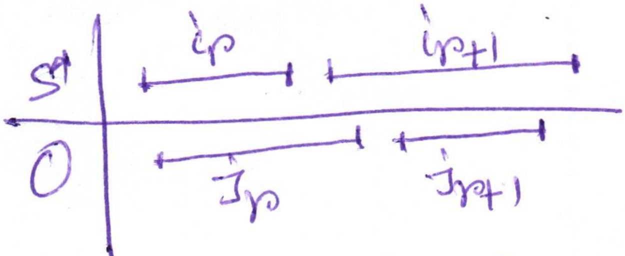
$\forall\, 1 \le l \le r \quad f(i_l) \le f(j_l)$

IS: $f(i_{r+1}) \le f(j_{r+1})$

↳ Pf. by contradiction

Assume for contradiction $f(i_{r+1}) > f(j_{r+1})$

By IH. $f(i_r) \le f(j_r)$



↳ $i_{r+1} \in R$
$j_{r+1} \in R$

Consider the greedy algo right after i_r is added to S.

$\Rightarrow$ As $f(j_{r+1}) < f(i_{r+1})$ greedy algo cannot pick i_{r+1}

$\Rightarrow$ Contradicts $i_{r+1} \in S^*$

$\square$