Lecture 19

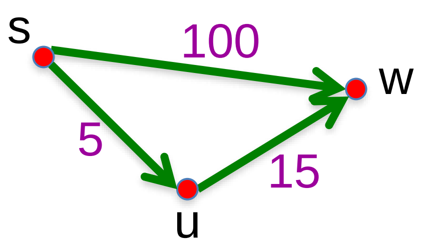
CSE 331

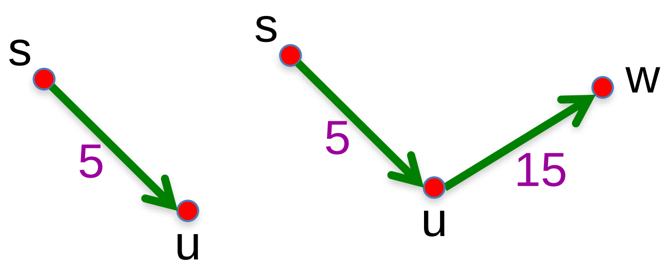
Shortest Path problem

Input: *Directed* graph G=(V,E)

Edge lengths, l_e for e in E

"start" vertex s in V



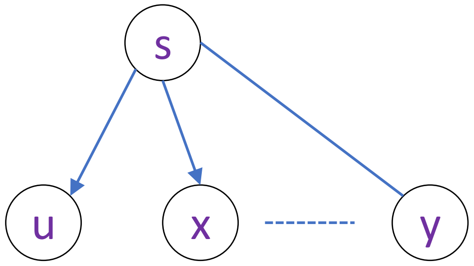


Output: Length of shortest paths from s to all nodes in V

Towards Dijkstra's algo: part one

Determine d(t) one by one

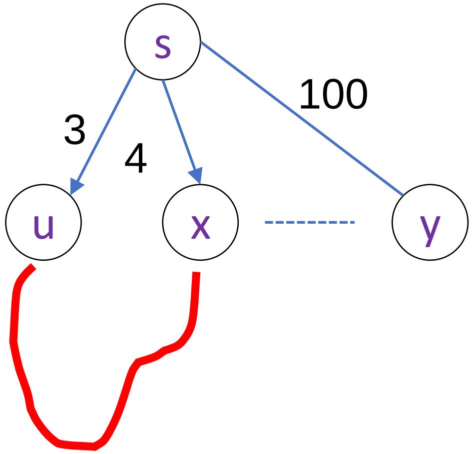
d(s) = 0



Towards Dijkstra's algo: part two

Determine d(t) one by one

Let u be a neighbor of s with smallest $l_{(s,u)}$



$$d(u) = l_{(s,u)}$$

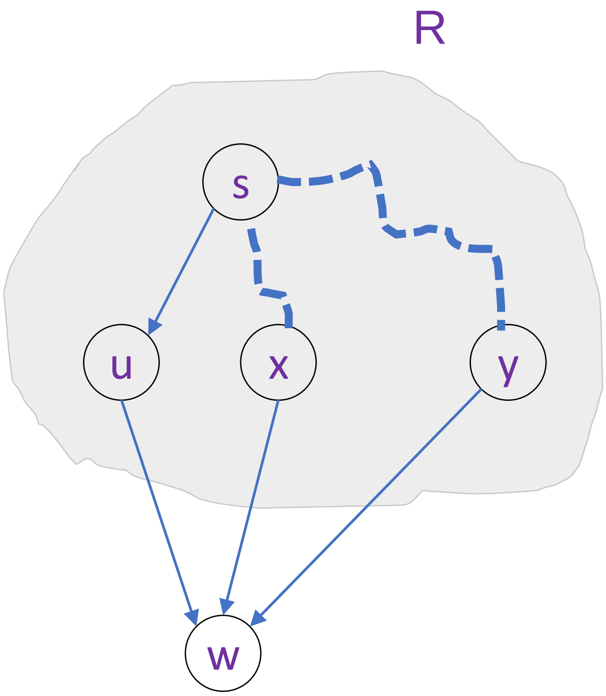
Length of ~ is ≥ 0

Not making any claim
on other vertices

Towards Dijkstra's algo: part three

Determine d(t) one by one

Assume we know d(v) for every v in R



Compute an upper bound d'(w) for every w not in R

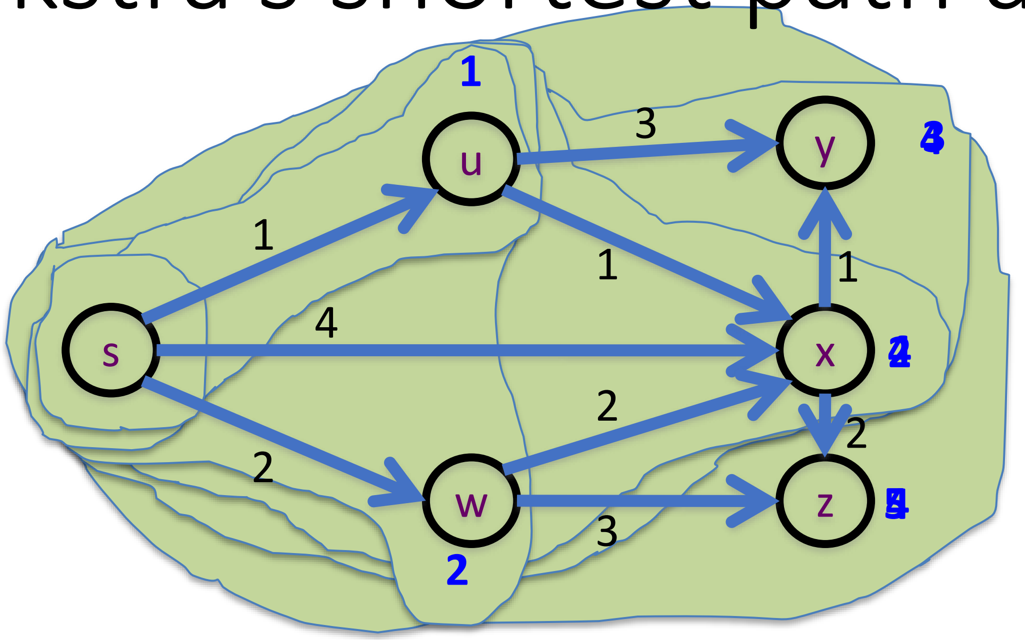
$$d(w) \leq d(u) + l_{(u,w)}$$

$$d(w) \leq d(x) + l_{(x,w)}$$

$$d(w) \leq d(y) + l_{(y,w)}$$

$$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$$

Dijkstra's shortest path algorithm



$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

d(s) = 0 d(u) = 1

d(w) = 2 d(x) = 2

d(y) = 3 d(z) = 4

Input: Directed G=(V,E), $l_e \geq 0$, s in V

R = {s}, d(s) =0

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

Add w to R

d(w) = d'(w)

Shortest paths