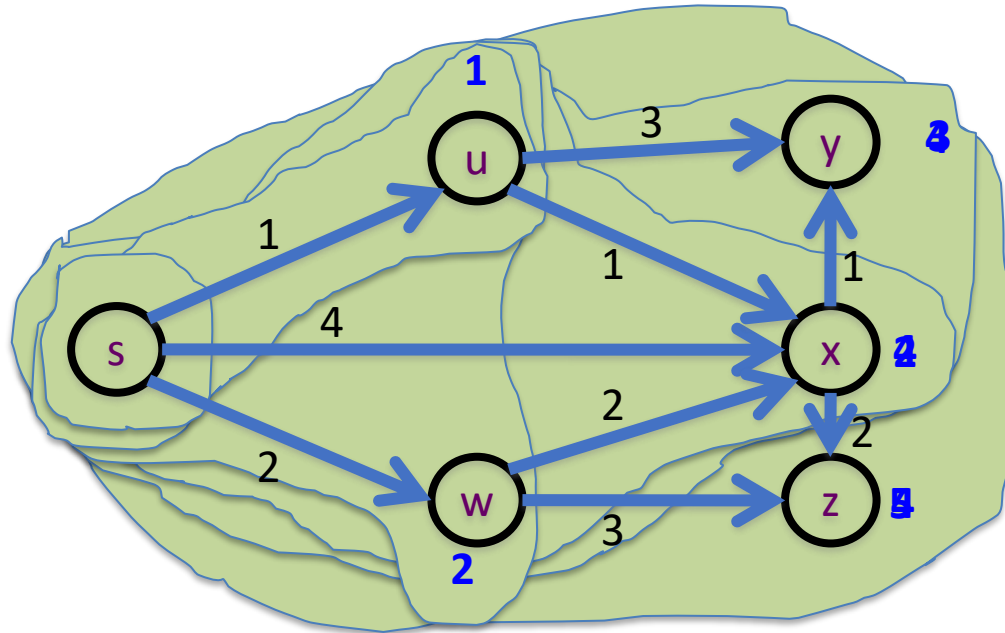


Lecture 20

CSE 331

Dijkstra's shortest path algorithm



Input: Directed $G=(V,E)$, $l_e \geq 0$, $s \in V$

$R = \{s\}$, $d(s) = 0$

While there is a x not in R with $(u,x) \in E$, $u \in R$

Pick w that minimizes $d'(w)$

Add w to R

$d(w) = d'(w)$

$$d'(w) = \min_{e=(u,w) \in E, u \in R} d(u) + l_e$$

$d(s) = 0$

$d(u) = 1$

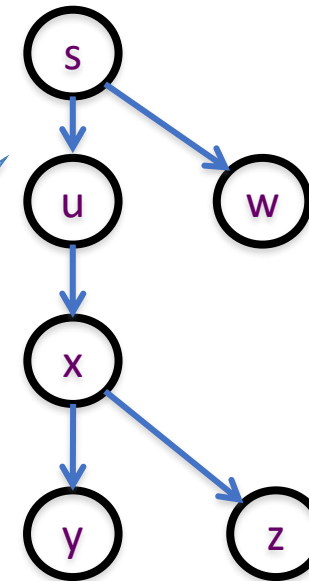
$d(w) = 2$

$d(x) = 2$

$d(y) = 3$

$d(z) = 4$

Shortest paths



Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain “shortest path tree” separately

Dijkstra's algorithm does not work with **negative** weights

Left as an exercise

Rest of Today's agenda

Prove the correctness of Dijkstra's Algorithm

Dijkstra's shortest path algorithm

P_u shortest s - u path in "Dijkstra tree"

$$d'(w) = \min_{e=(u,w) \text{ in } E, u \text{ in } R} d(u) + l_e$$

Input: Directed $G=(V,E)$, $l_e \geq 0$, s in V

$R = \{s\}$, $d(s) = 0$

While there is a x not in R with (u,x) in E , u in R

Pick w that minimizes $d'(w)$

Add w to R

$d(w) = d'(w)$

Lemma 1: At end of each iteration, if u in R , then P_u is a shortest s - u path

Lemma 2: If u is connected to s , then u in R at the end

Proof idea of Lemma 1

Dijkstra's shortest path algorithm

$$d'(w) = \min_{e=(u,w) \text{ in } E, u \text{ in } R} d(u) + l_e$$

Input: Directed $G=(V,E)$, $l_e \geq 0$, $s \text{ in } V$

$R = \{s\}$, $d(s) = 0$

While there is a x not in R with $(u,x) \text{ in } E$, $u \text{ in } R$

Pick w that minimizes $d'(w)$

Add w to R

$d(w) = d'(w)$

At most n
iterations

$$\sum_{x \in V} O(\ln_x + 1) \\ = O(m+n) \text{ time}$$

$O((m+n)n)$ time bound is trivial

$O((m+n) \log n)$ time implementation with priority Q

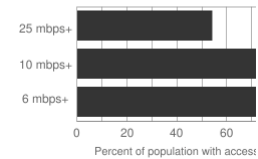
Reading Assignment

Sec 4.4 of [KT]

Make broadband more available

Cattaraugus County

Population: 79518
Median Income: \$41,368.88
Access to any cable technology: 67.5%
Access to two or more wireline providers: 61.2%

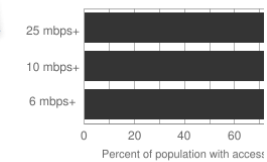


Say you are tasked to come up with the infrastructure

BOTH
technical and societal issues

Erie County

Population: 913295
Median Income: \$49,817.67
Access to any cable technology: 98.9%
Access to two or more wireline providers: 96.8%



Building a fiber network

Lay down fibers to connect n locations

All n locations should be connected

Laying down a fiber costs money



What is the cheapest way to lay down the fibers?

Today's agenda

Minimum Spanning Tree (MST) Problem

Greedy algorithm(s) for MST problem

On to the board...