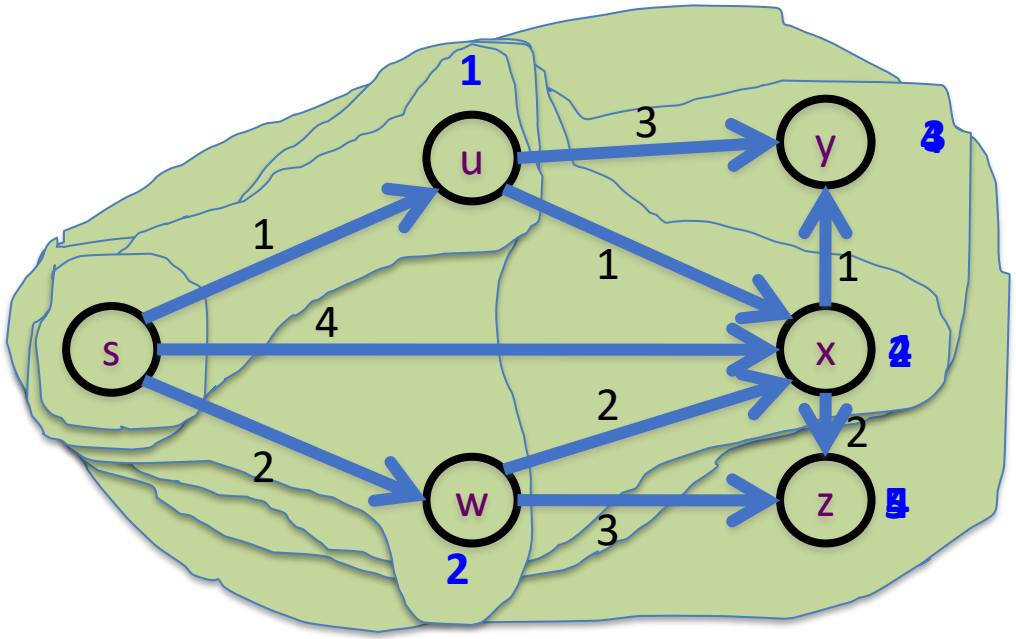
Lecture 20

CSE 331

Dijkstra's shortest path algorithm



Input: Directed G=(V,E), $l_e \geq 0$, s in V

R = {s}, d(s) =0

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

Add w to R

d(w) = d'(w)

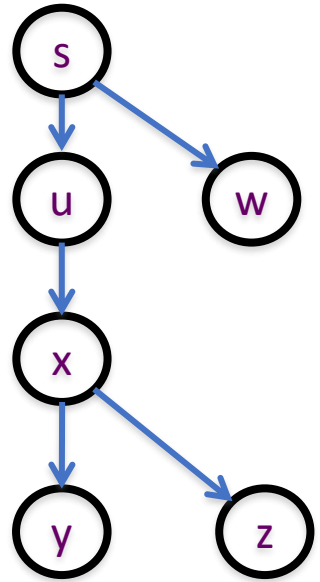
$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

d(s) = 0 d(u) = 1

d(w) = 2 d(x) = 2

d(y) = 3 d(z) = 4

Shortest paths



Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain "shortest path tree" separately

Dijkstra's algorithm does not work with negative weights

Left as an exercise

Rest of Today’s agenda

Prove the correctness of Dijkstra’s Algorithm

Dijkstra's shortest path algorithm

P_u shortest s-u path in "Dijkstra tree"

$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

Input: Directed $G=(V,E)$, $l_e \geq 0$, s in V

$R = \{s\}$, $d(s) = 0$

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes $d'(w)$

Add w to R

$d(w) = d'(w)$

Lemma 1: At end of each iteration, if u in R, then P_u is a shortest s-u path

Lemma 2: If u is connected to s, then u in R at the end

Proof idea of Lemma 1

Dijkstra's shortest path algorithm

$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

Input: Directed $G=(V,E)$, $l_e \geq 0$, s in V

$R = \{s\}$, $d(s) = 0$

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes $d'(w)$
Add w to R
$d(w) = d'(w)$

At most n iterations

$\Sigma_{x \in V} O(\ln_x+1)$
$=O(m+n)$ time

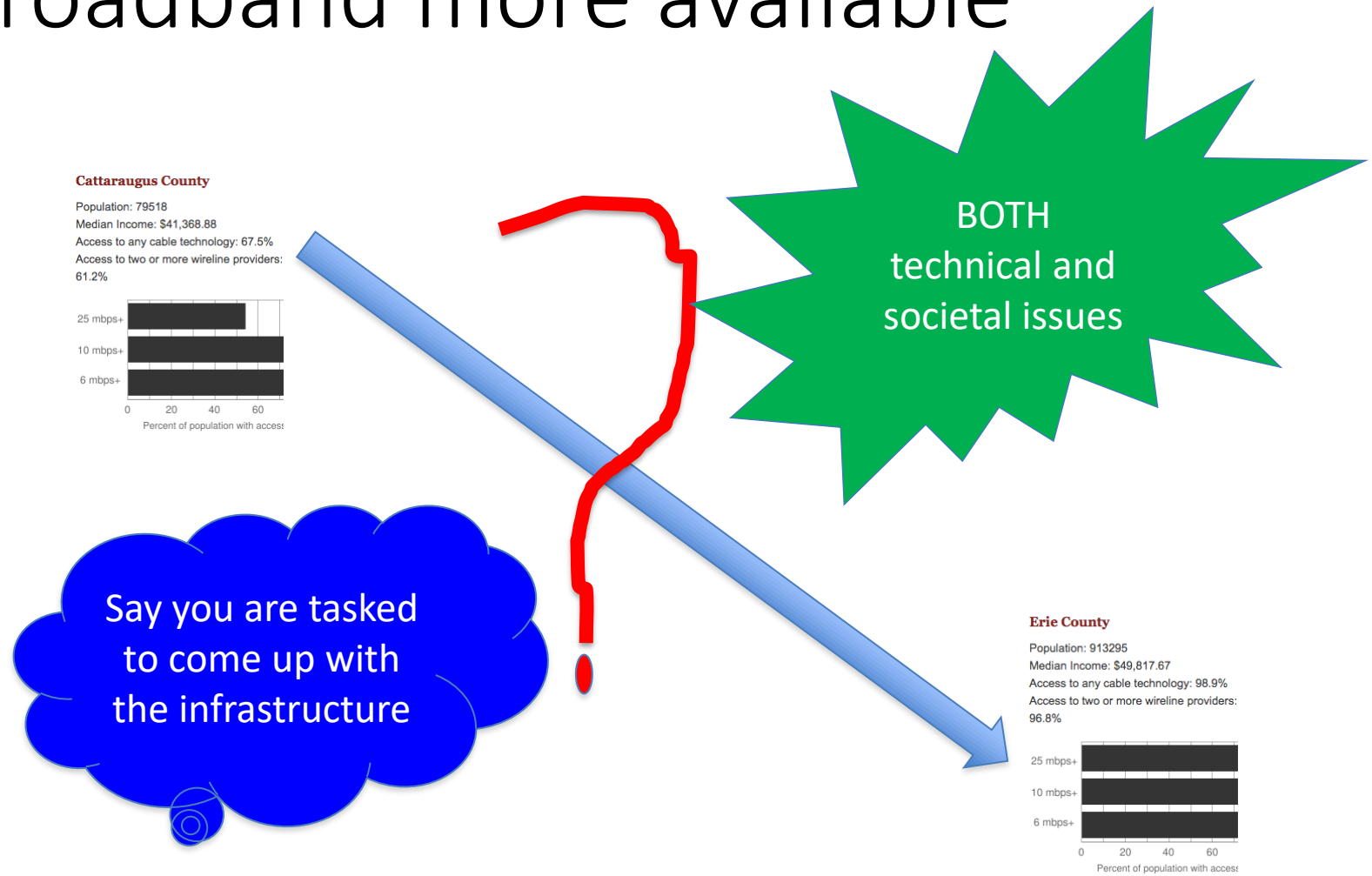
$O((m+n)n)$ time bound is trivial

$O((m+n) \log n)$ time implementation with priority Q

Reading Assignment

Sec 4.4 of [KT]

Make broadband more available



Building a fiber network

Lay down fibers to connect n locations

All n locations should be connected

Laying down a fiber costs money

What is the cheapest way to lay down the fibers?

Today’s agenda

Minimum Spanning Tree (MST) Problem

Greedy algorithm(s) for MST problem

On to the board…