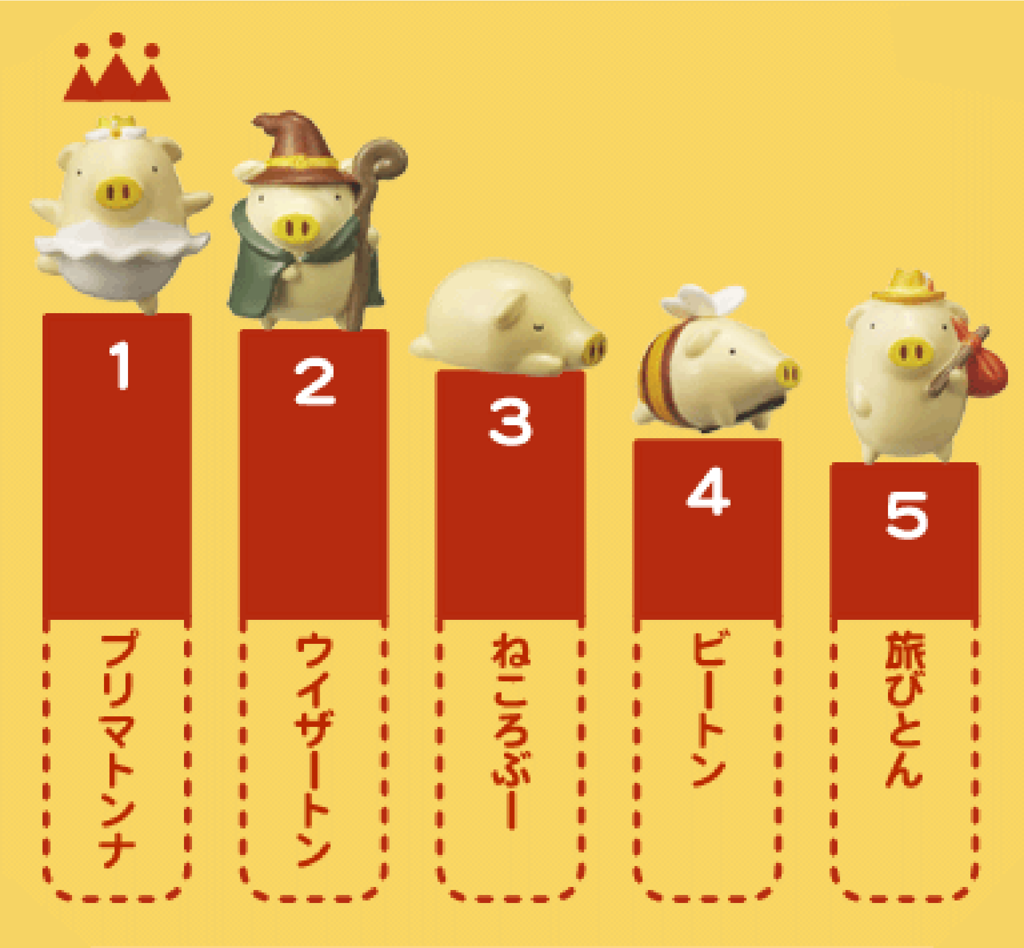
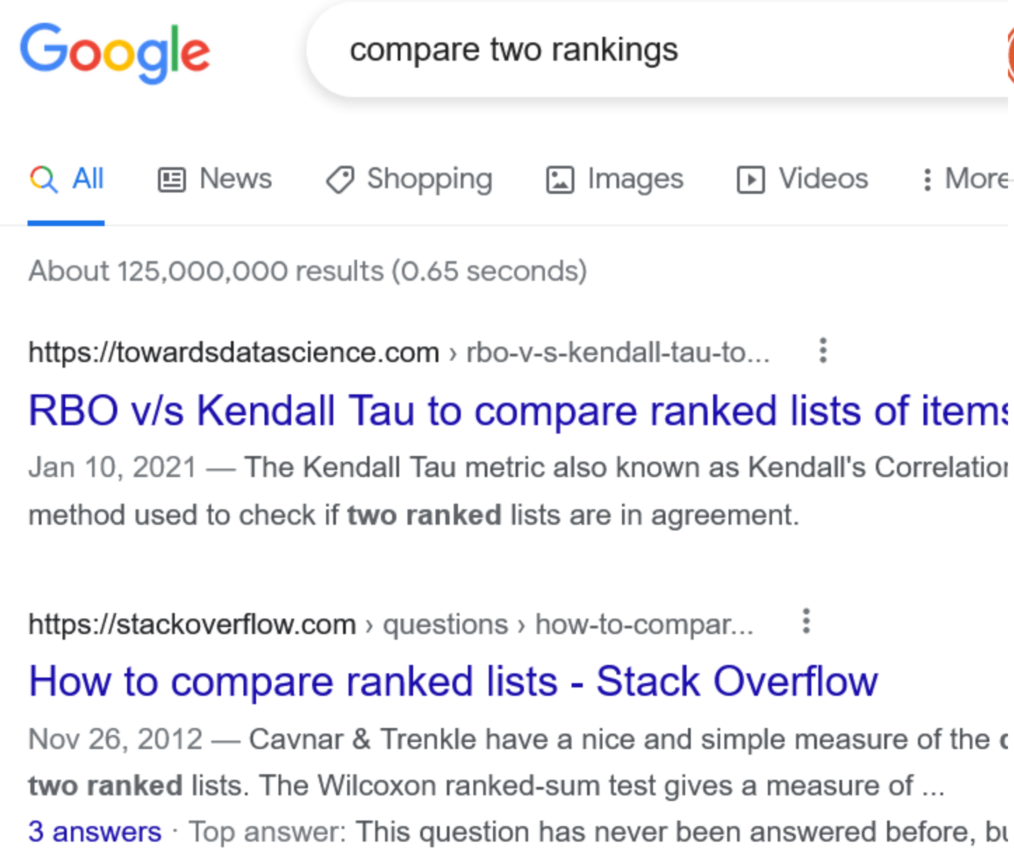
Lecture 25

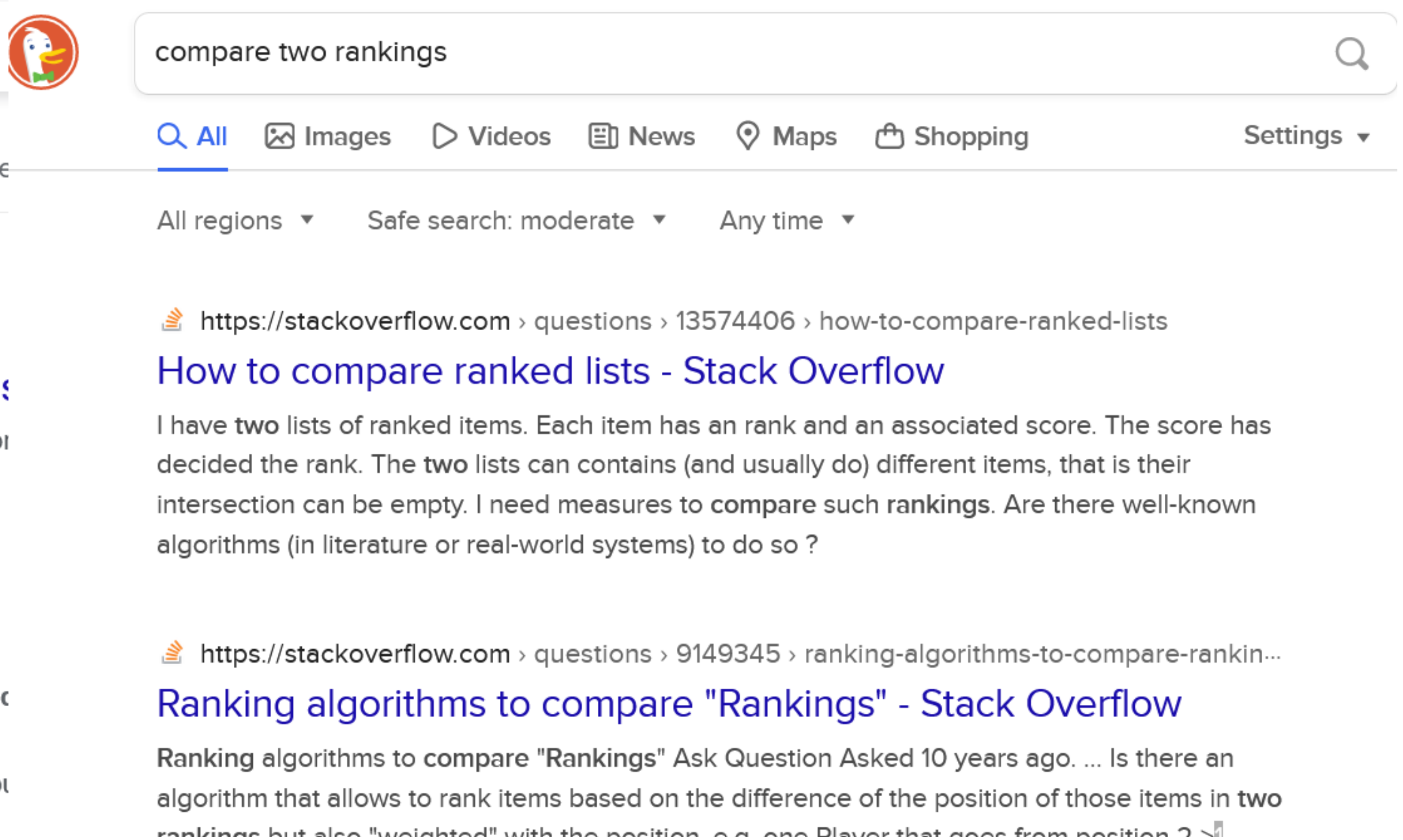
CSE 331

Rankings



How close are two rankings?





(A Very Simple) Collaborative Filtering Example

Each user: a ranking of movies/shows on Netflix.

Assumption: Each user ranks all movies/shows on Netflix.

Hypothesis: A user is close to another user if their rankings are close.

1. Movie-X 2. Movie-Y 3. Movie-Z

User 1	User 2	User 3
1	3	1
2	2	3
3	1	2

Rankings

Problem Formulation

Input: A ranking $a_1, \ldots, a_i, a_j, \ldots a_n$. (i.e., a permutation of 1, 2, …, n)

Implicit assumption: 1, 2, …, n is the "true" ranking (i.e., you compare other rankings to this ranking).

Output: The number of inversions.

Inversion: (i, j) is an inversion if

1. $i < j$ AND 2. $a_i > a_j$

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User 2: how many inversions?
Answer: every pair is an inversion.

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Number of inversions = $\binom{3}{2}$ = 3, inversions = {(1, 2), (1, 3), (2, 3)}.

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User 1: How many inversions?
Answer: one inversion: (2, 3).

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Rankings

<u>Example 2:</u>
A = (1, 2, …, n).
How many inversions? 0
If $a_1, \dots, a_i, a_j, \dots a_n$ are sorted, then no inversions.

<u>Example 3:</u>
A = (n, …, 1).
How many inversions? $\binom{n}{2}$

$$0 \leq \# \text{ inversions} \leq \binom{n}{2}$$

Solve a harder problem

Input: $a_1, .., a_n$

Output: LIST of all inversions

```
L = ϕ
for i in 1 to n-1
   for j in i+1 to n
       If a_i > a_j
           add (i,j) to L

return  L
```

Optimal for the listing problem

Example 1: All inversions-- (2i-1,2i)

2	1	3	4	6	5	7	8

Only check (i,i+1) pairs

Q1: Solve listing problem in O(n) time?

Q2: Recursive divide and conquer algorithm to count the number of inversions?

CountInv (a,n)

if $n = 1$ return 0

if $n = 2$ return $a_1 > a_2$

$a_L = a_1 , .., a_{[n/2]}$

$a_R = a_{[n/2]+1} , .., a_n$

return CountInv(a_L, [n/2]) + CountInv(a_R, n- [n/2])

Can be horribly wrong in general

CountInv (a,n)

if $n = 1$ return 0

if $n = 2$ return $a_1 > a_2$

$a_L = a_1 , .., a_{[n/2]}$

$a_R = a_{[n/2]+1} , .., a_n$

return CountInv(a_L, $[n/2]$) + CountInv(a_R, $n- [n/2]$)

Example where instance has non-zero (can be $\Omega(n^2)$) inversions and algo returns 0?

5	6	1	2

All 4 “crossing” pairs are inversions

Bad case: “crossing inversions”

CountInv (a,n)

if $n = 1$ return 0

if $n = 2$ return $a_1 > a_2$

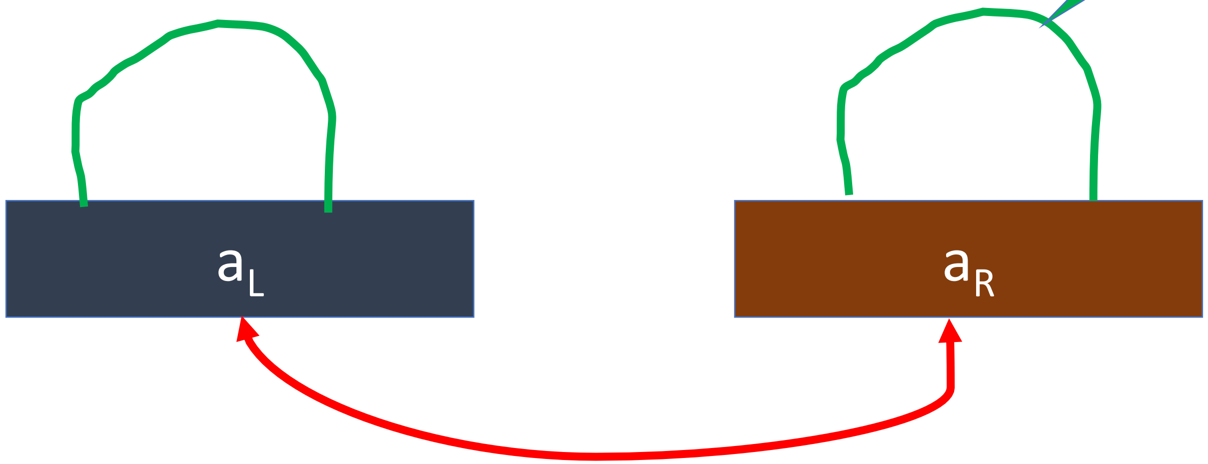
$a_L = a_1 , .., a_{[n/2]}$

$a_R = a_{[n/2]+1} , .., a_n$

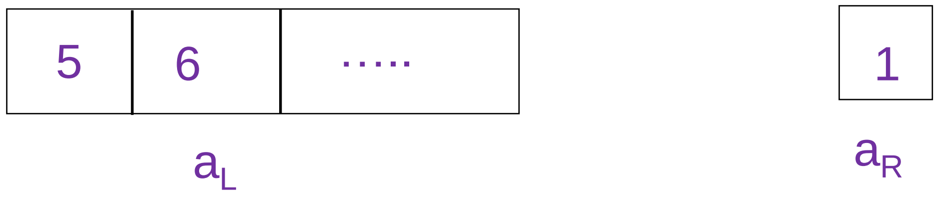
return CountInv(a_L, $[n/2]$) + CountInv(a_R, $n - [n/2]$)

Yes!

Are a_L and a_R sorted?



Example 2: Solving the bad case



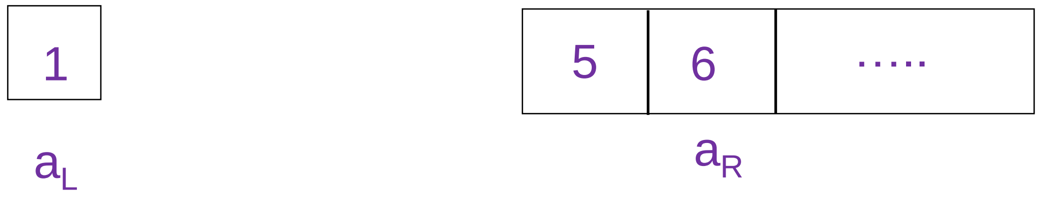
a_L is sorted

First element is a_L is larger than first/only element in a_R

O(1) algorithm to count number of inversions?

return size of a_L

Example 3: Solving the bad case



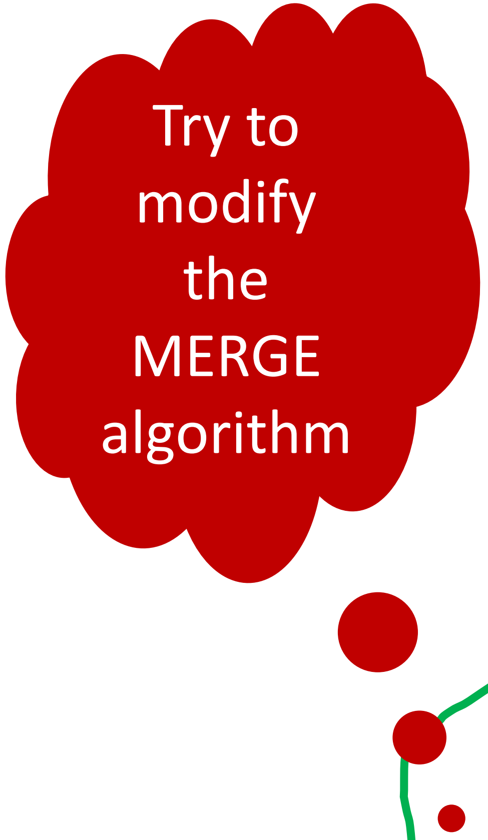
a_R is sorted

First/only element is a_L is smaller than first element in a_R

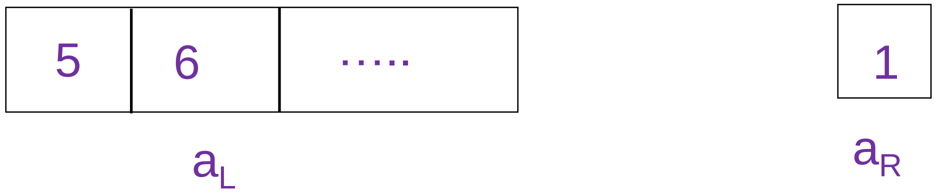
O(1) algorithm to count number of inversions?

return 0

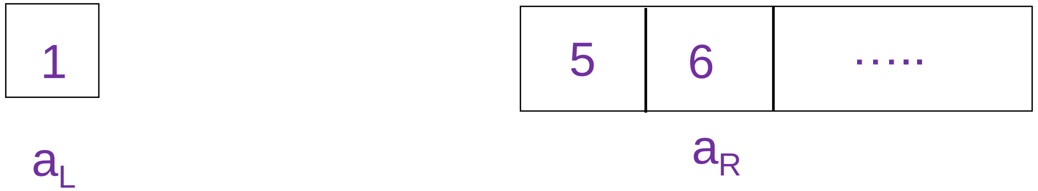
Solving the bad case

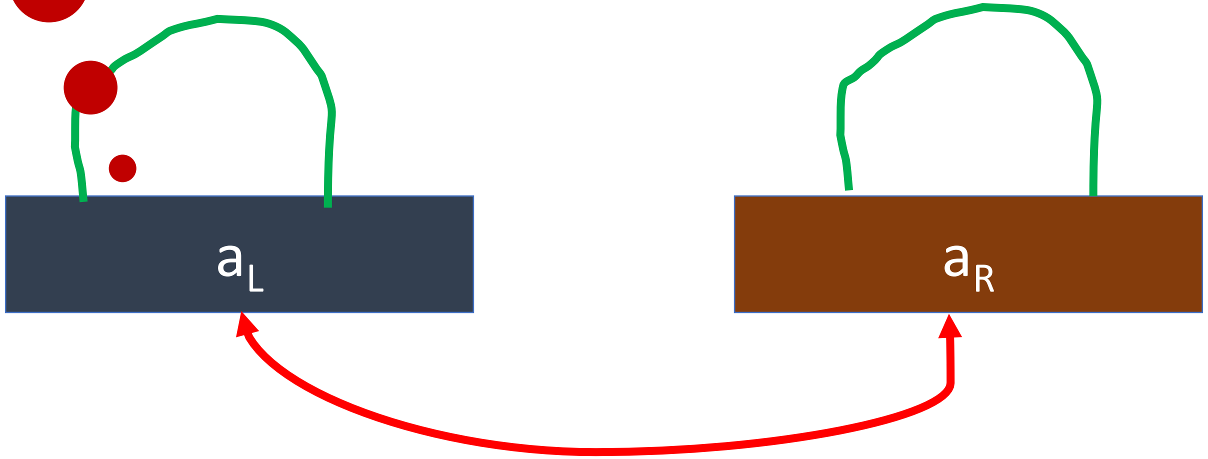


First element of a_L is larger than first element of a_R



First element of a_L is smaller than first element of a_R





Divide and Conquer

Divide up the problem into at least two sub-problems

Solve all sub-problems: Mergesort

Recursively solve the sub-problems

Solve stronger sub-problems: Inversions

“Patch up” the solutions to the sub-problems for the final solution

MergeSortCount algorithm

Input: $a_1, a_2, \ldots, a_n$

Output: Numbers in sorted order+ #inversion

MergeSortCount(a, n)

If n = 1 **return** (0 , a_1)

If n = 2 **return** (a1 > a2, min(a_1,a_2); max(a_1,a_2))

$a_L = a_1, \ldots, a_{n/2}$ $a_R = a_{n/2+1}, \ldots, a_n$

(c_L, a_L) = MergeSortCount(a_L, n/2)

(c_R, a_R) = MergeSortCount(a_R, n/2)

(c, a) = MERGE-COUNT(a_L,a_R)

return ($c+c_L+c_R$,a)

Counts #crossing-inversions+ MERGE