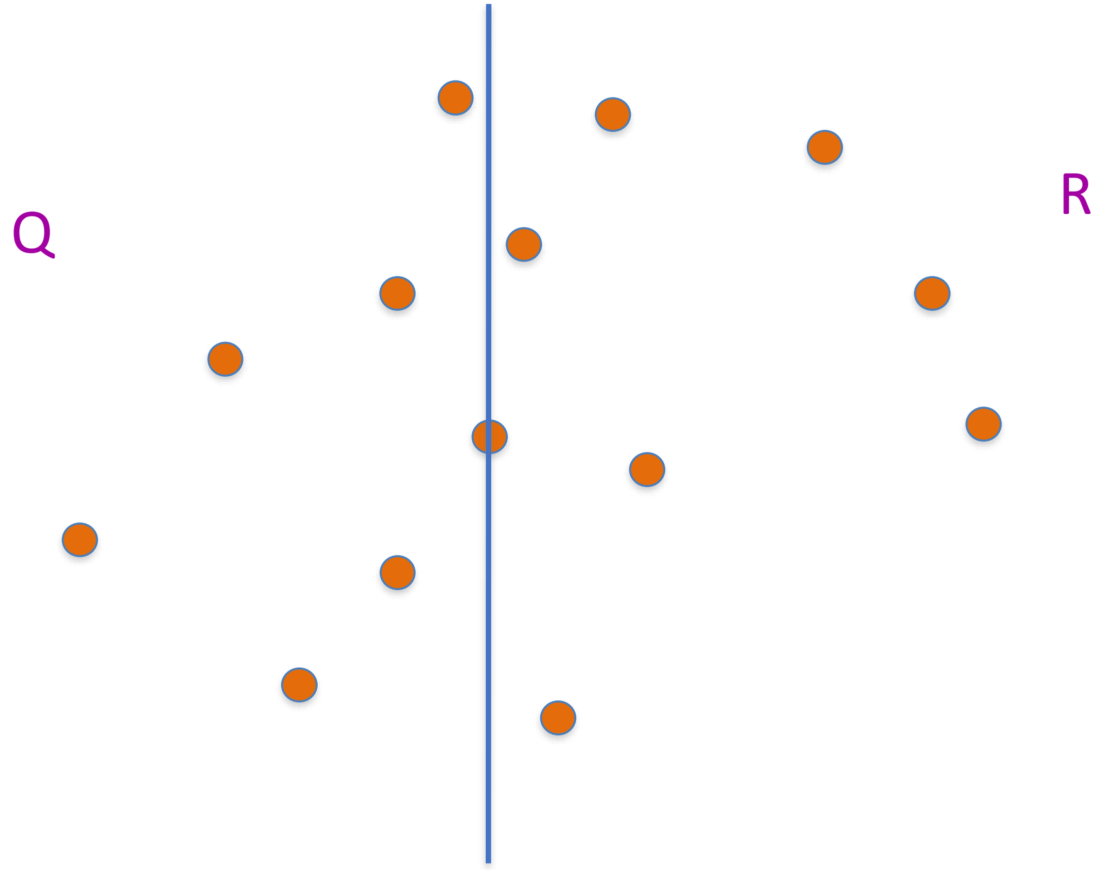
Lecture 28

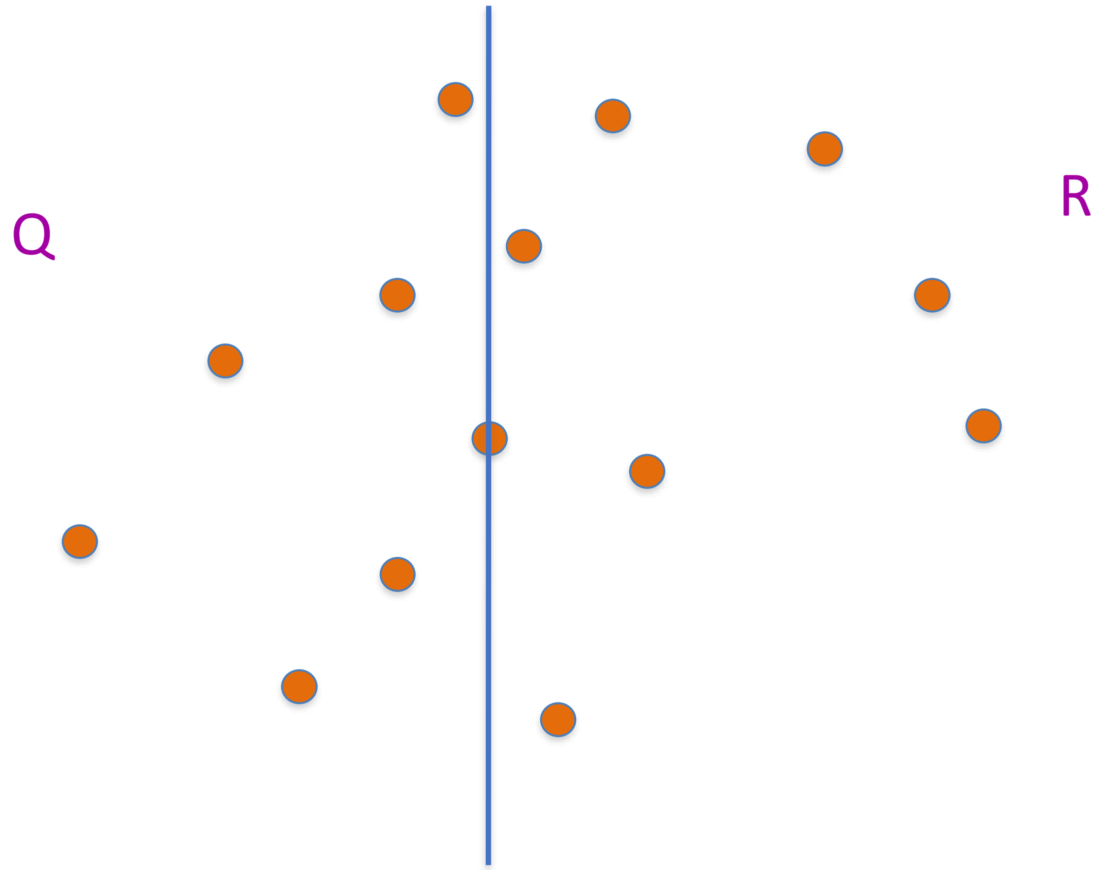
CSE 331

Dividing up P



First n/2 points according to the x-coord

Recursively find closest pairs



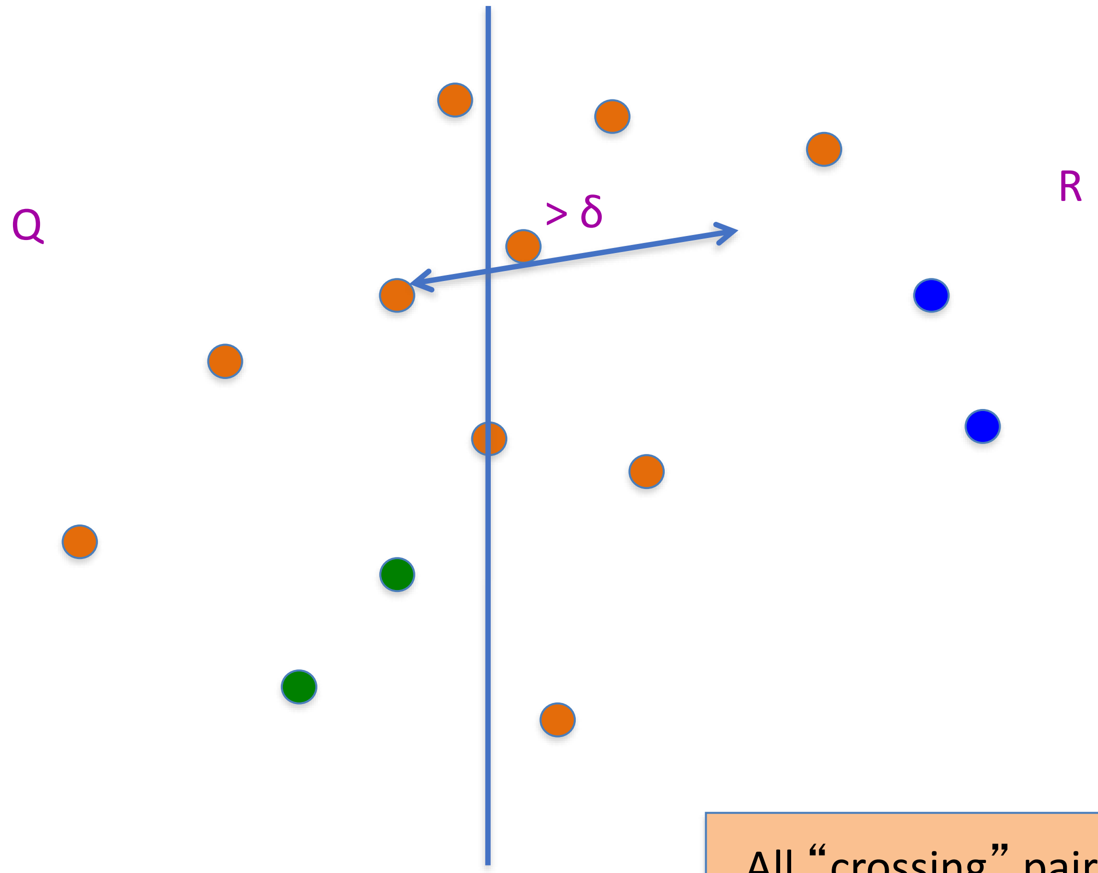
δ = min (**blue**, **green**)

An aside: maintain sorted lists

P_x and P_y are P sorted by x-coord and y-coord

Q_x, Q_y, R_x, R_y can be computed from P_x and P_y in $O(n)$ time

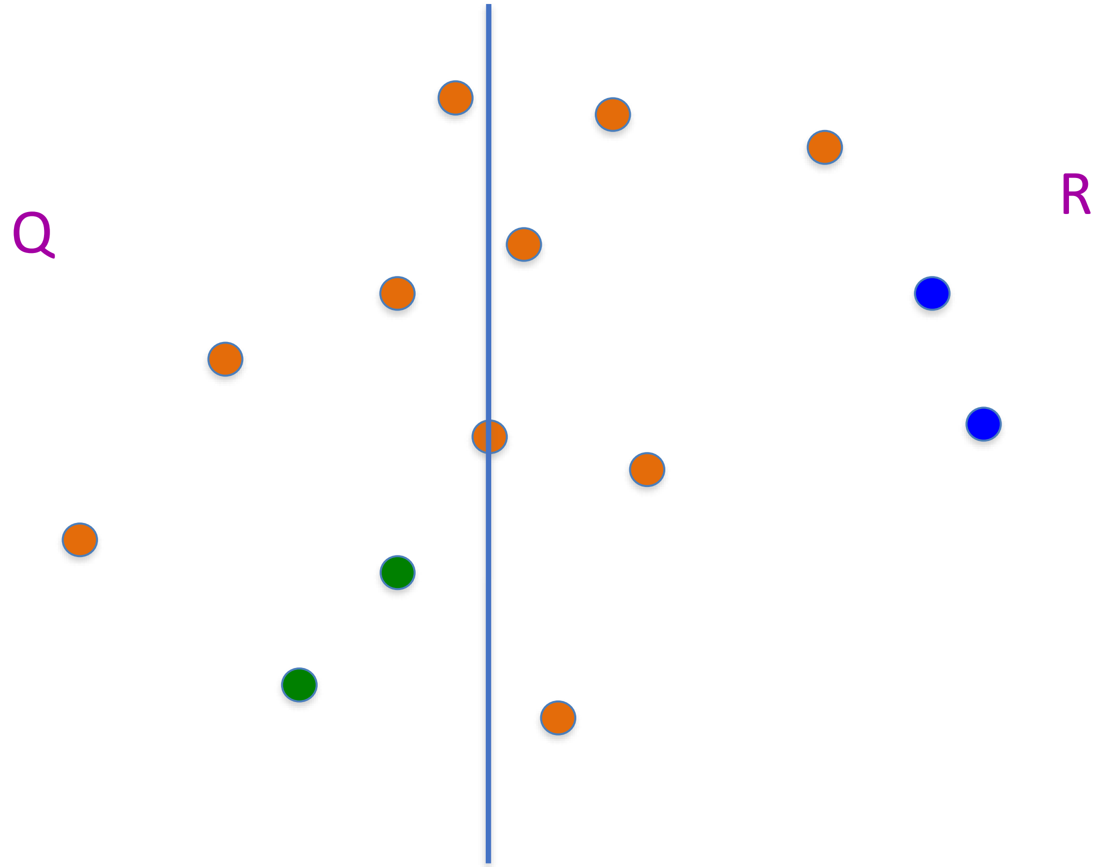
An easy case



All "crossing" pairs have distance > δ

δ = min (**blue**, **green**)

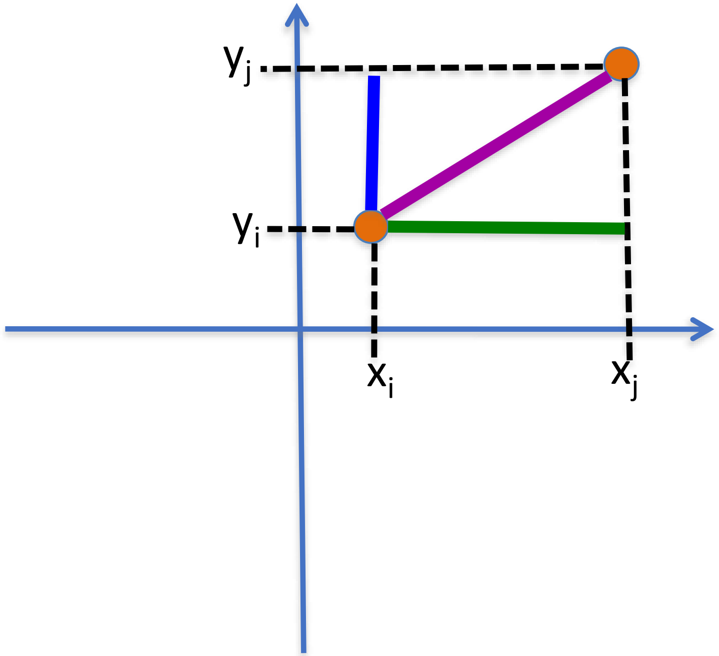
Life is not so easy though



δ = min (blue, green)

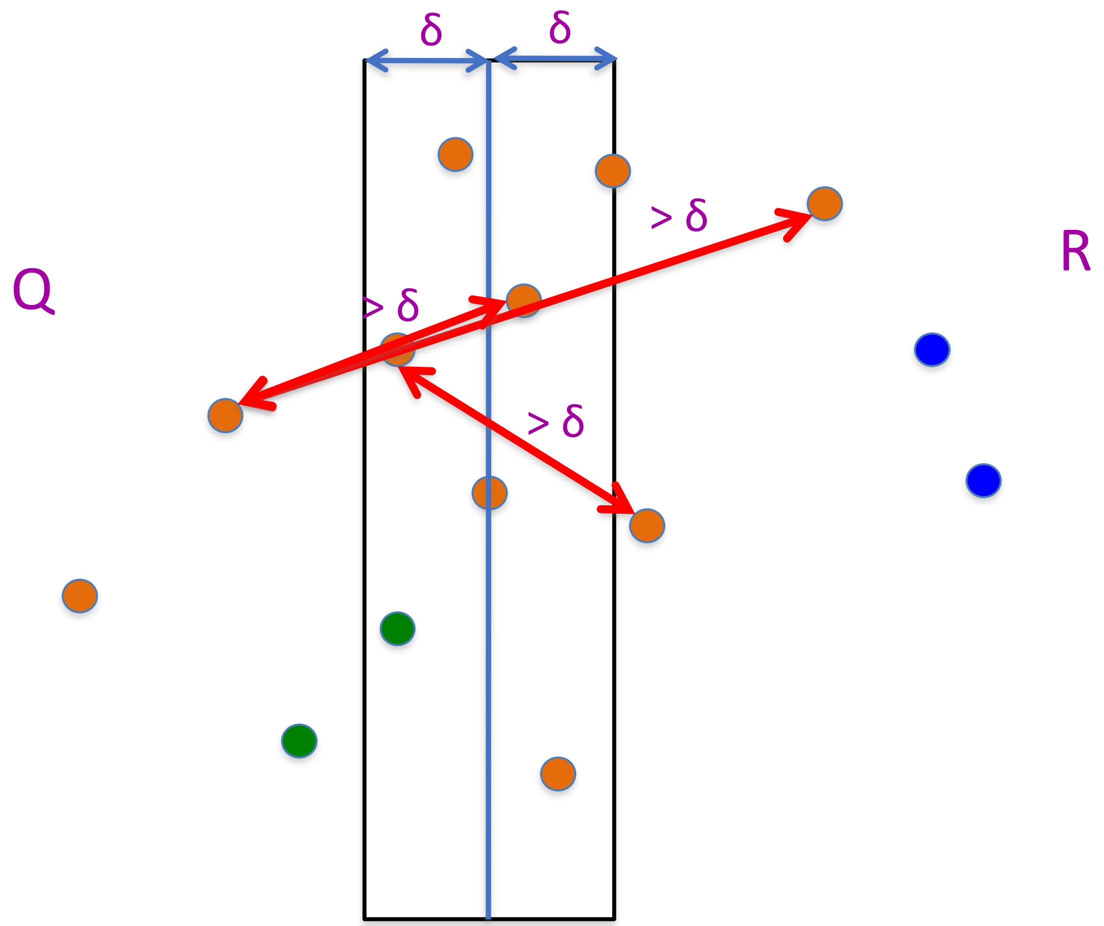
Euclid to the rescue (?)

$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$



The distance is larger than the x or y-coord difference

Life is not so easy though



δ = min (blue, green)

All we have to do now

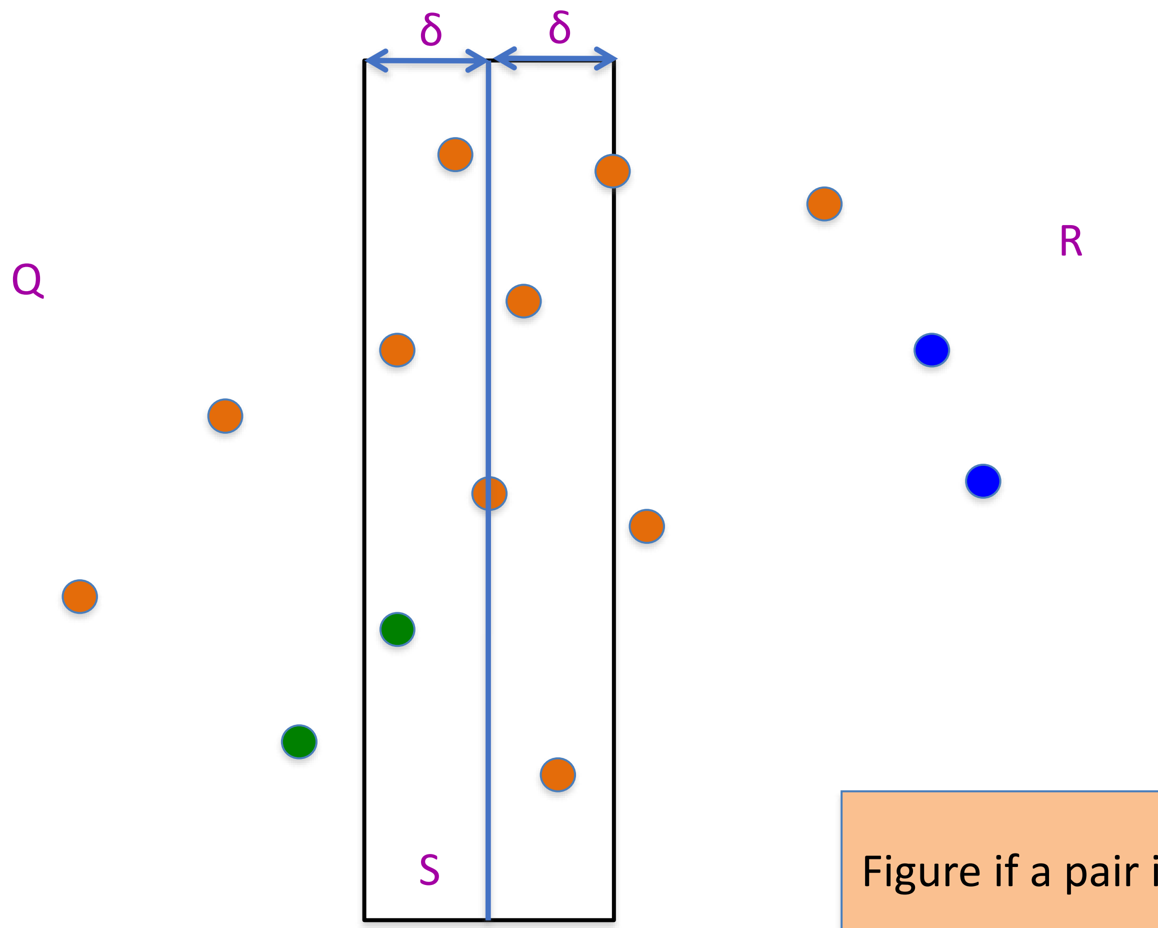


Figure if a pair in S has distance < δ

δ = min (blue, green)

The algorithm so far…

$O(n \log n) + T(n)$

Input: n 2-D points $P = \{p_1,\ldots,p_n\}$; $p_i=(x_i,y_i)$

Sort P to get P_x and P_y — $O(n \log n)$

Closest-Pair (P_x, P_y)

- If $n < 4$ then find closest point by brute-force
- Q is first half of P_x and R is the rest — $O(n)$
- Compute Q_x, Q_y, R_x and R_y — $O(n)$
- (q_0,q_1) = Closest-Pair (Q_x, Q_y)
- (r_0,r_1) = Closest-Pair (R_x, R_y)
- $\delta = \min(\, d(q_0,q_1), d(r_0,r_1)\,)$ — $O(n)$
- S = points (x,y) in P s.t. $|x - x^*| < \delta$ — $O(n)$
- return Closest-in-box $(S, (q_0,q_1), (r_0,r_1))$ — Assume can be done in $O(n)$

$T(< 4) = c$

$T(n) = 2T(n/2) + cn$

$O(n \log n)$ overall

Rest of today's agenda

Implement Closest-in-box in $O(n)$ time