# Lecture 31

CSE 331

# Weighted Interval Scheduling

Input: $n$ jobs $(s_i, f_i, v_i)$

Output: A schedule $S$ s.t. no two jobs in $S$ have a conflict

Goal: max $\Sigma_{i \text{ in } S} v_j$

Assume: jobs are sorted by their finish time

# Couple more definitions

p(j) = largest i < j s.t. i does not conflict with j

= 0 if no such i exists

p(j) < j

OPT(j) = optimal value on instance 1,..,j

<u>Note:</u>

1. p(1), …, p(n) can be computed in O(n log n) time. [Ex]
2. Any algo to computer p(1), …, p(n) needs to make Ω(n log n ) comparisons. [Ex]

# Property of OPT

j in OPT(j)

j not in OPT(j)

$$OPT(j) = \max \{ v_j + OPT(p(j)), OPT(j-1) \}$$

Given OPT(1),…., OPT(j-1),
how can one figure out if j
in optimal solution or not?

# A recursive algorithm

Compute-Opt(j)

Correct for j=0

Proof of correctness by induction on j

If j = 0 then return 0

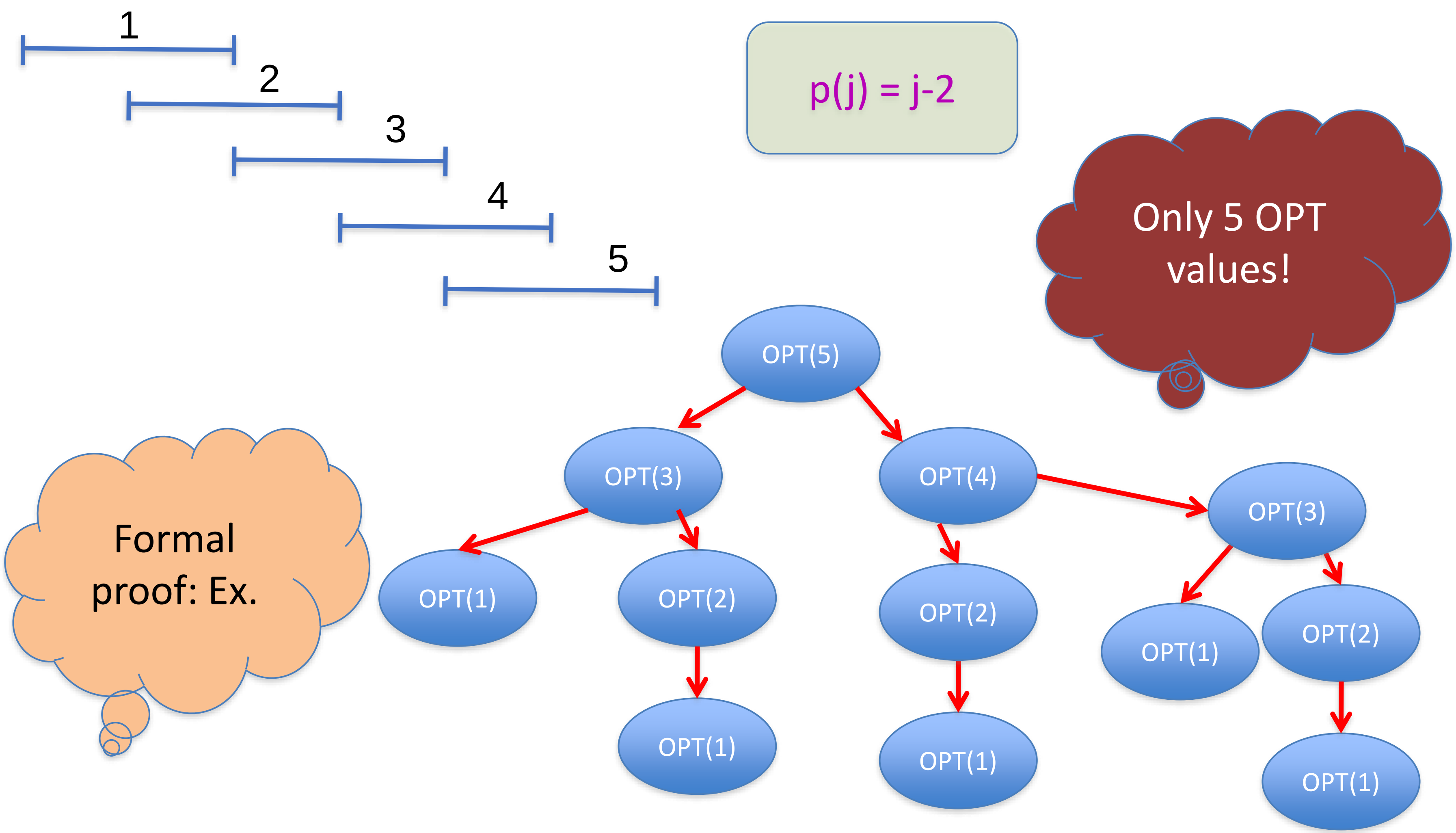
return max { $v_j$ + Compute-Opt( p(j) ), Compute-Opt( j-1 ) }

= OPT( p(j) )

= OPT( j-1 )

$$OPT(j) = \max \{ v_j + OPT( p(j) ), OPT(j-1) \}$$

# Exponential Running Time



# Using Memory to be smarter

Using more space can reduce runtime!

# How many distinct OPT values?

# A recursive algorithm

M-Compute-Opt(j) = OPT(j)

M-Compute-Opt(j)

If j = 0 then return 0

If M[j] is not null then return M[j]

M[j] = max { $v_j$ + M-Compute-Opt( p(j) ), M-Compute-Opt( j-1 ) }

return M[j]

Run time = O(# recursive calls)

# Bounding # recursions

M-Compute-Opt(j)

If j = 0 then return 0

If M[j] is not null then return M[j]

M[j] = max { $v_j$ + M-Compute-Opt( p(j) ), M-Compute-Opt( j-1 ) }

return M[j]

O(n) overall

Whenever a recursive call is made an M value is assigned

At most n values of M can be assigned

# Property of OPT

$$OPT(j) = \max \{ v_j + OPT(p(j)), OPT(j-1) \}$$

Given OPT(1), …, OPT(j-1),
one can compute OPT(j)

# Recursion+ memory = Iteration

Iteratively compute the OPT(j) values

Iterative-Compute-Opt

$M[0] = 0$

For $j=1,\ldots,n$

$M[j] = \max \{ v_j + M[p(j)], M[j-1] \}$

$M[j] = OPT(j)$

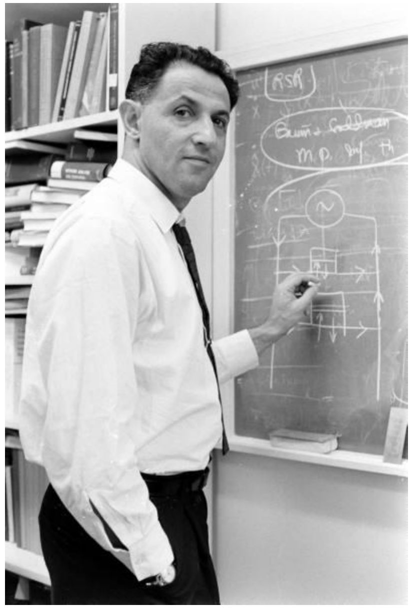
$O(n)$ run time

# Algo run on the board…

# Reading Assignment

Sec 6.1, 6.2 of [KT]

# When to use Dynamic Programming


Richard Bellman

There are polynomially many sub-problems

$\text{OPT}(1), \ldots, \text{OPT}(n)$

Optimal solution can be computed from solutions to sub-problems

$$\text{OPT}(j) = \max \{ v_j + \text{OPT}(\, p(j) \,), \text{OPT}(j-1) \}$$

There is an ordering among sub-problem that allows for iterative solution

$\text{OPT}(j)$ only depends on $\text{OPT}(j-1), \ldots, \text{OPT}(1)$

# Scheduling to min idle cycles

$n$ jobs, $i^{th}$ job takes $w_i$ cycles

You have $W$ cycles on the cloud

What is the maximum number of cycles you can schedule?

# Subset sum problem

Input: $n$ integers $w_1, w_2, \ldots, w_n$

bound $W$

Output: subset $S$ of $[n]$ such that

(1) sum of $w_i$ for all i in $S$ is at most $W$

(2) $w(S)$ is maximized