Lecture 4

CSE 331

Read the syllabus CAREFULLY!

Syllabus Quiz

Admin Options

CA Options

Options

- View handin history
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- Download handout

Due: **May 15th 2023, 11:59 pm**

Last day to handin: **May 15th 2023, 11:59 pm**

No graded material will be handed back till you pass the syllabus quiz!

Office hours (OH) finalized

Actions

TA Office Hour Schedule

Hi all,

We have finalized TA office hours. An (i) after a TA name means an *in-person* office hour and a (v) after a TA name means that the office hour is virtual-only on zoom. This same information can be found in our syllabus.

All in-person TA office hours that do not mention a specific location in the list below will be in Salvador lounge. Locations may change in the third week. Please keep an eye on this post and check this post to know the correct location before you go to a TA office hour.

- Mondays
    - 9-10am, Rrucha (i)
    - 1-2pm, Zachary (i)
    - 2-3pm, Korey (i)

First Week OH

- TAs will discuss the following
  - Proof by Induction, Proof by Contradiction
  - Proof idea, proof details
- Go ask you proof questions!

Separate Proof idea/proof details

</> Note

Notice how the solution below is divided into proof idea and proof details part. **THIS IS IMPORTANT: IF YOU DO NOT PRESENT A PROOF IDEA, YOU WILL NOT GET ANY CREDIT EVEN IF YOUR PROOF DETAILS ARE CORRECT**.

Proof Idea

As the hint suggests there are two ways of solving this problem. (I'm presenting both the solutions but of course you only need to present one.)

We begin with the approach of reducing the given problem to a problem you have seen earlier. [2] Build the following complete binary tree: every internal node in the tree represents a "parent" RapidGrower while its two children are the two RapidGrowers it divides itself into. After s seconds this tree will have height s and the number of RapidGrowers in the container after s seconds is the number of leaf nodes these complete binary tree has, which we know is 2^s. Hence, the claim is correct.

The proof by induction might be somewhat simpler for this problem if you are not comfortable with reduction. In this case let $R(s)$ be the number of RapidGrowers after s seconds. Then we use induction to prove that $R(s) = 2^s$ while using the fact that $2 \cdot 2^s = 2^{s+1}$.

Proof Details

We first present the reduction based proof. Consider the complete binary tree with height s and call it $T(s)$. Further, note that one can construct $T(s+1)$ from $T(s)$ by attaching two children nodes to all the leaves in $T(s)$. Notice that the newly added children are the leaves of $T(s+1)$. Now assign the root of $T(0)$ as the original RapidGrower in the container. Further, for any internal node in $T(s)$ ($s \geq 0$), assign its two children to the two RapidGrowers it divides itself into. Then note that there is a one to one correspondence between the RapidGrowers after s seconds and the leaves of $T(s)$. [3] Then we use the well-known fact (cite your 191/250 book here with the exact place where one can find this fact): $T(s)$ has 2^s leaves, which means that the number of RapidGrowers in the container after s seconds is 2^s, which means that the claim is correct.

Solutions to HW 0 out

CSE 331 Syllabus Piazza Schedule Homeworks Autolab Project Support Pages channel

HW 0
Soln 0

Allowed Sources
Homework Policies

Solutions mework 0

Please note that we will provi ne course webpage for HW 0 **only**. From HW 1 onwards, we will only hand out links to a PDF with the solutions on piazza.

What is a proof?

The goal of this question is to present a gentle start to proofs. In particular, the idea is to highlight a common mistake students make while writing proofs.

The Problem

Consider the following "proof":

Questions/Comments?

Incorrect Proof Details: Q1(b) on HW0

Argument does not use ANYTHING about the problem statement!

Follows from part (a)

…of perfect matchings with n men and n wo…

This assumes number of perfect matchings only depends on n

Base case: $P(1) = 1! = 1$

Inductive hypothesis: Assume that $P(n-1) = (n-1)!$

Inductive step: Note that $P(n) = n*P(n-1) = n*(n-1)! = n!$

What are the issues with the above “proof”?

Incorrect Proof Details: Q1(b) on HW0

Claim 1: Number of perfect matchings is = number of permutations of 1…n

Needs justification

Claim 2: Number of permutations of 1…n is n!

Needs justification

Follow from 191 (?)

Claims 1 + 2 prove the result

What are the issues with the above proof?

Proof by contradiction for Q1(a)

Assume for contradiction there is an example where number of perfect matchings depends on the identities of the men and women.

You can only assume things about the example directly implied by it being a counter-example

Let $n = 1$ and consider two cases

(1) M = {BP} and W = {JA}

(2) M = {BBT} and W = {AJ}

In both cases the number of perfect matchings is $1 = 1$!

Hence contradiction.

There is NO contradiction

What are the issues with the above proof?

Matching Employers & Applicants

Input: Set of employers (E)
Set of applicants (A)
Preferences

Output: An assignment of applicants to employers that is "stable"

For every x in A and y in E such that x is **not** assigned to y, either

(i) y prefers *every* accepted applicant to x; or

(ii) x prefers her employer to y

Questions to think about

1) How do we specify preferences? **Preference lists**

2) Ratio of applicant vs employers **1:1**

3) Formally what is an assignment? **(perfect) matching**

4) Can an employer get assigned > 1 applicant? **NO**

5) Can an applicant have > 1 job? **NO**

6) How many employer/applicants in an applicants/employers preferences? **All of them**

7) Can an employer have 0 assigned applicants? **NO**

8) Can an applicant have 0 jobs? **NO**

Stable Marriage Problem

n men

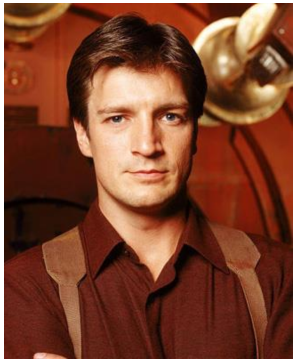
Each with a preference list

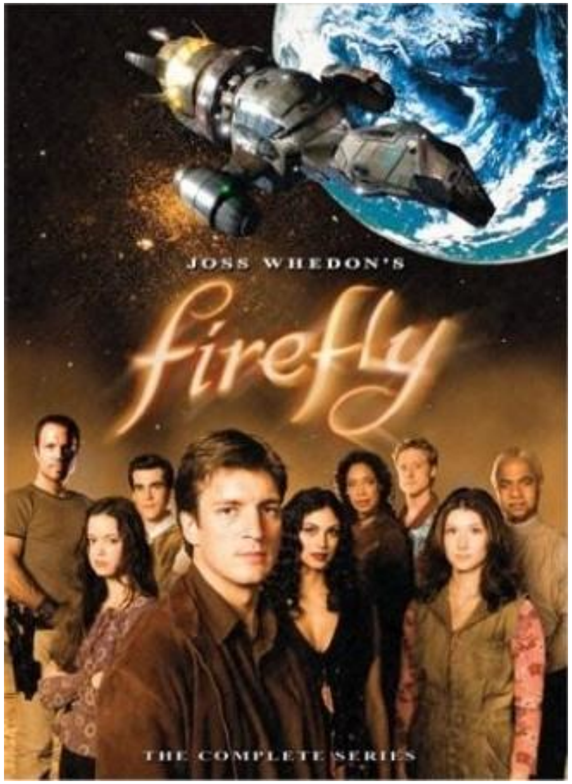
n women

Match/marry them in a “stable” way

On matchings

Mal



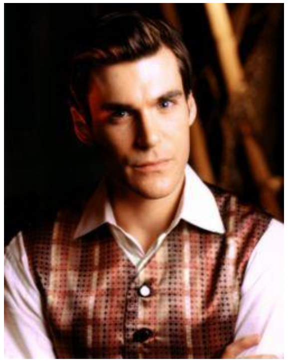


Inara

Wash

Zoe

Simon



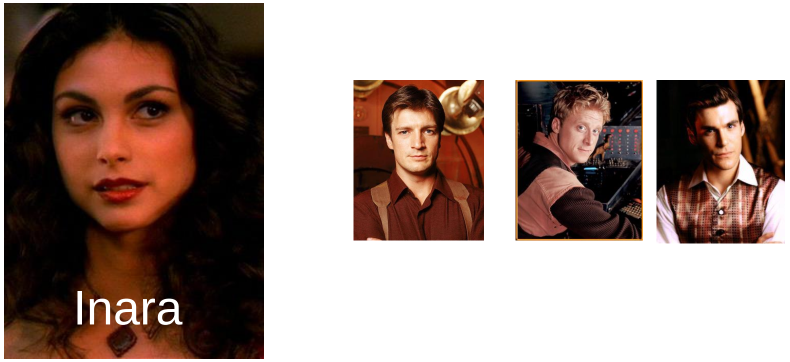
Kaylee

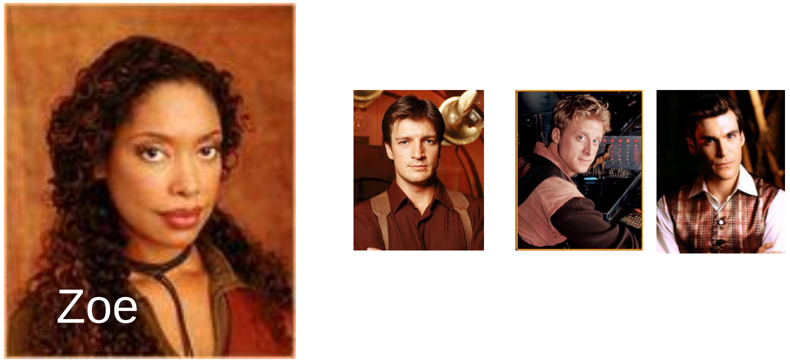
A valid matching

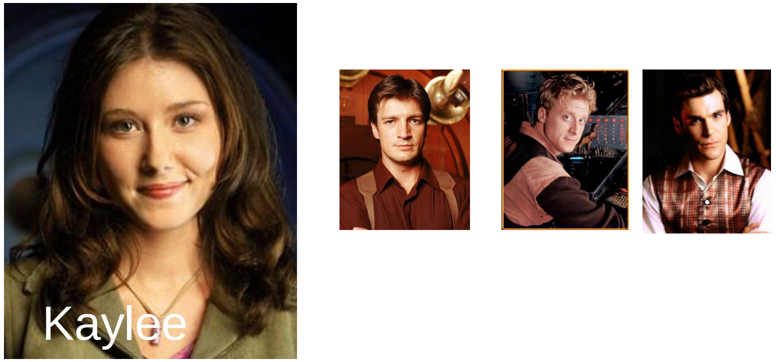
Not a matching

Perfect Matching

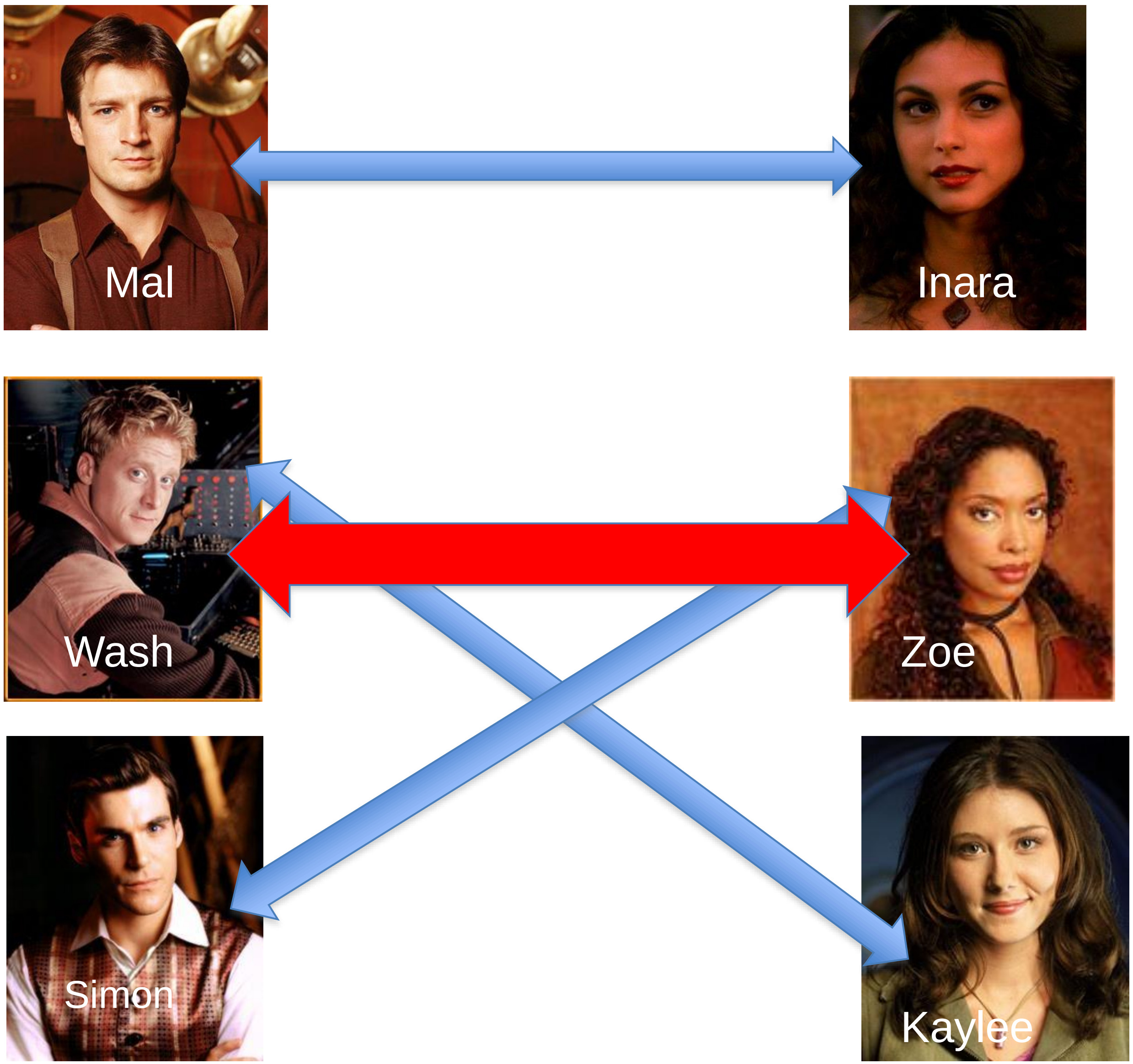
Preferences







Instability



Work things out on paper