Lecture 5

CSE 331

A Note

Discomfort with proofs

Proof basics will not be covered in class anymore

Please read support pages and some utilize (next few) Office hours!

Questions/Comments?

(Perfect) Matching

A matching $S \subseteq M \times W$ such that following conditions hold:

S is a **set** of pairs (m,w) where m in M and w in W

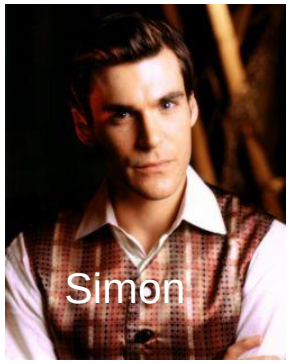
(1) For every woman w in W, exist *at most* one m such that (m,w) in S

(2) For every man m in M, exist *at most* one w such that (m,w) in S

Perfect matching

A valid matching

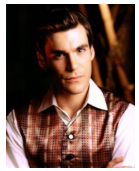
A valid matching

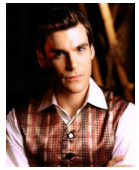


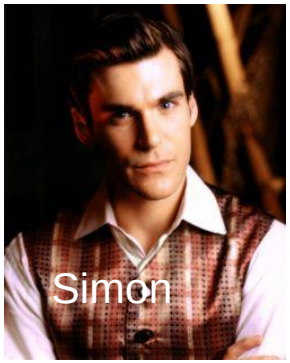
Not a matching

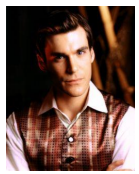
Perfect Matching

Preferences

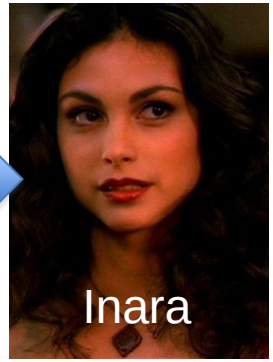


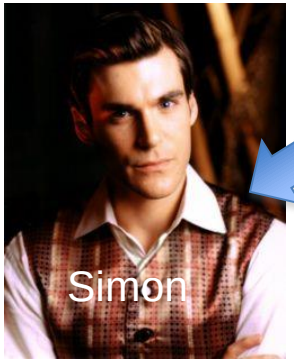


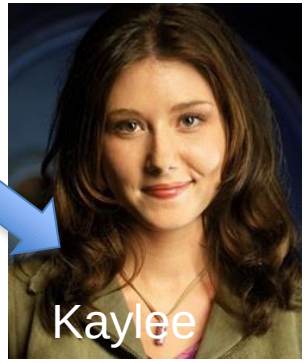




Instability







Work things out on paper

A stable marriage

Even though BBT and JA are not very happy

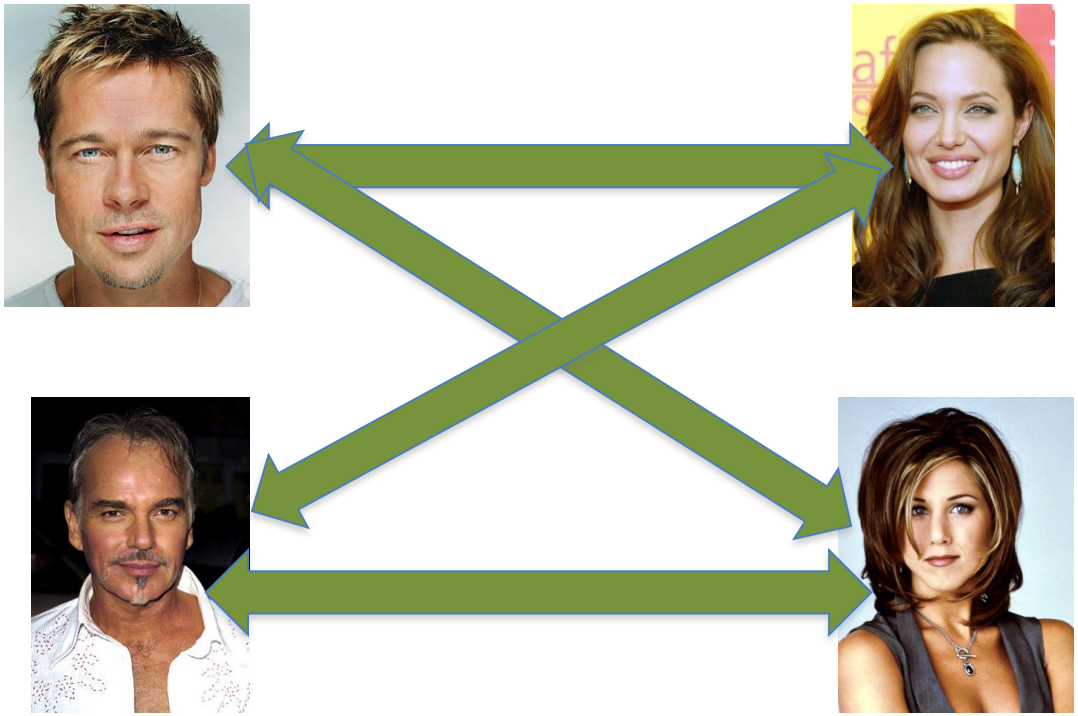
BP

AJ

BBT

JA

Two stable marriages



Stable Marriage problem

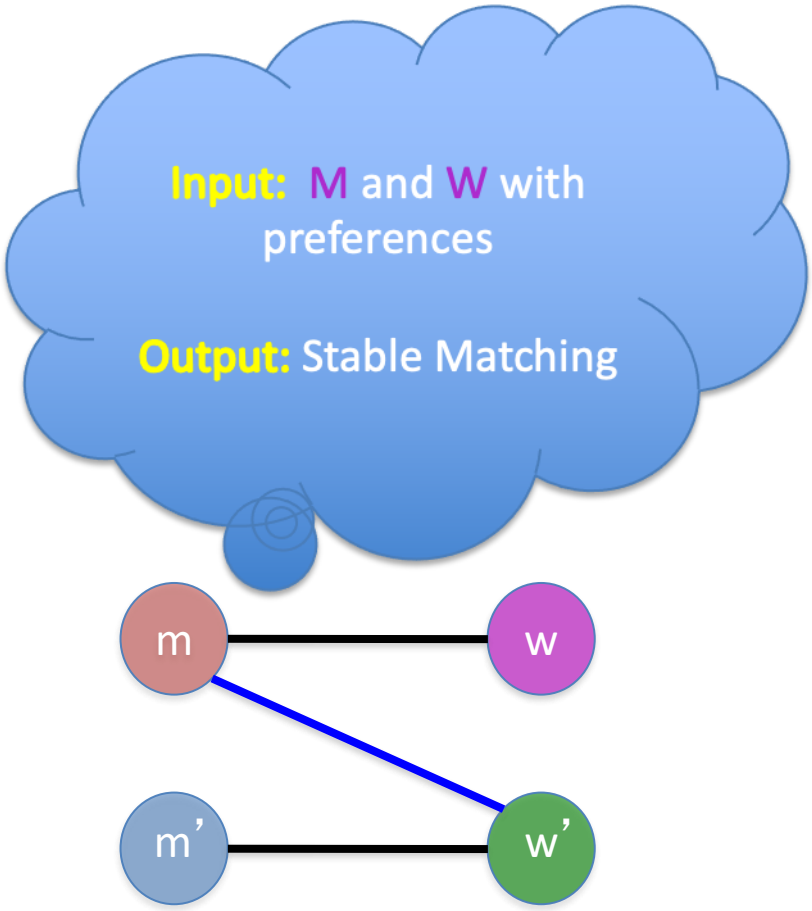
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instablity



Stable matching = perfect matching + no instablity

Questions/Comments?

Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

The naïve algorithm

Incremental algorithm to produce all n! prefect matchings?

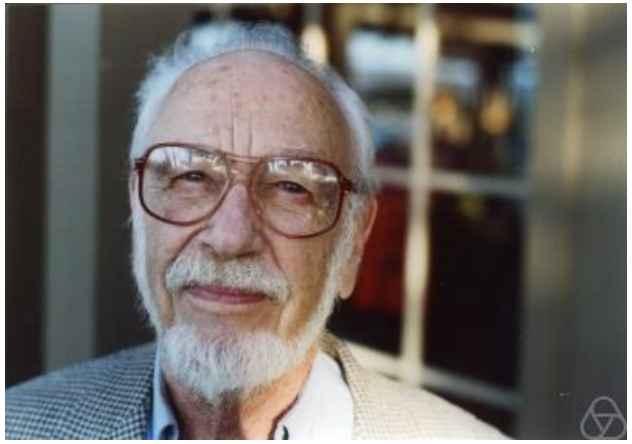
Go through all possible perfect matchings S

If S is a stable matching

then Stop

Else move to the next perfect matching

Gale-Shapley Algorithm

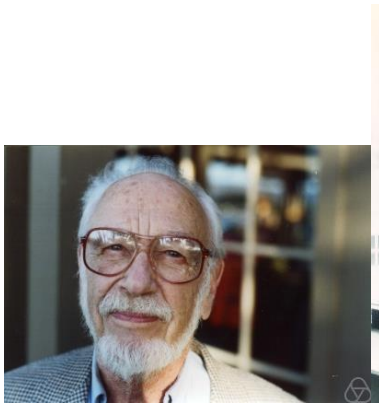


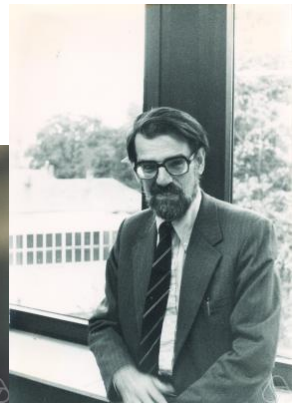
David Gale

Lloyd Shapley

$O(n^2)$ algorithm

Moral of the story…





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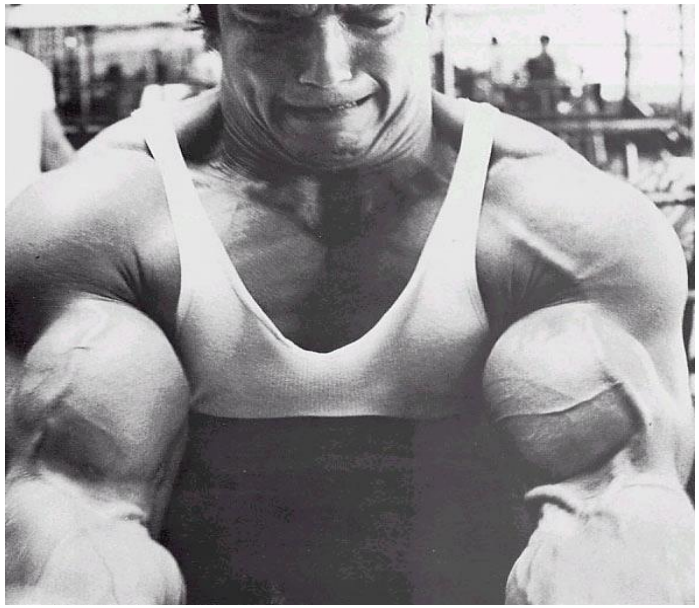

Questions/Comments?

Rest of today's agenda

Gale Shapley (GS) algorithm

Run of GS algorithm on an instance

Questions/Comments?