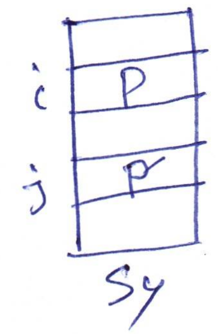
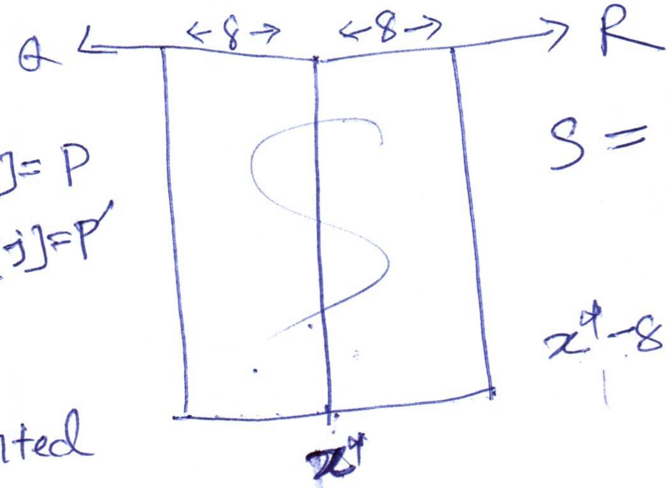
Apr 15



$S_y[i] = p$
$S_y[j] = p'$



$S = \{(x,y) \in P \mid |x - x^*| < \delta\}$

$x^* - \delta < x < x^* + \delta$

S_y: pts in S sorted by y-coord.

__Lemma (5.10, KT):__ For every $p \neq p' \in S$ s.t. $d(p,p') < \delta$, if $S_y[i] = p$, $S_y[j] = p'$,

then $|i - j| \leq 15$.

__Note:__ The value 15 can be as small as 9, or even 7.

* $O(n)$ algo. to compute closest-in-Box?

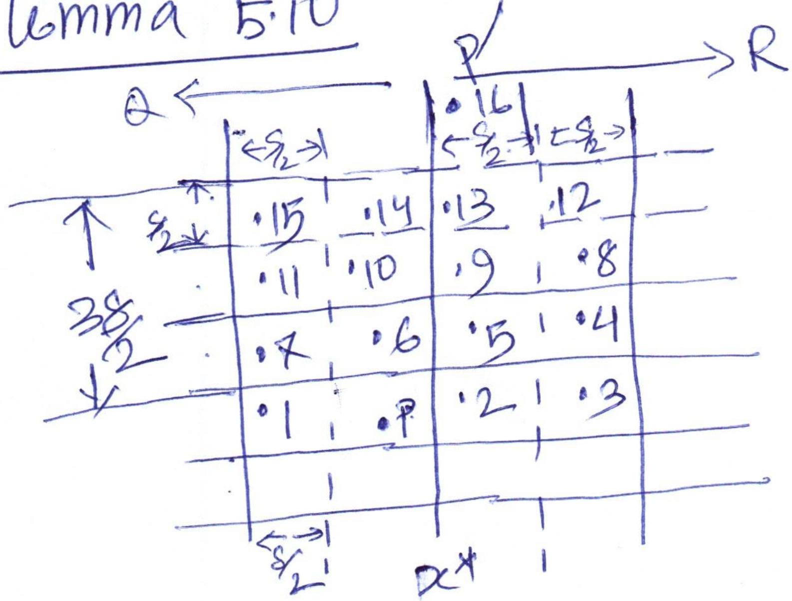
for $i = 1, \ldots, n-1$ — $O(n)$
- check $(S_y[i], S_y[i+1])$, $(S_y[i], S_y[i+2])$, ..., $(S_y[i], S_y[i+15])$ — $O(1)$
- let (p_i, p_i') closest pair of points

let (p, p') be the closest pair of points among (p_1, p_1'), (p_2, p_2'), ..., (p_{n-1}, p_{n-1}') — $O(n)$

if $d(p,p') < \delta$ return (p,p')
else return None — $O(1)$

Total runtime: $O(n)$

Pf (idea) of lemma 5.10



$d(P, P') \geq 3\delta/2$

$> \delta$

$\Rightarrow$ contradicts (*)

$\square$

Assume $d(P, P') < \delta$ (*)

$S_y[i] = P$

$S_y[j] = P'$

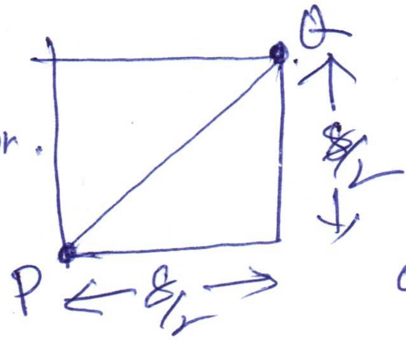
WLOG, $j \geq i+16$

$\underset{P}{i}, i+1, i+2, \dots \underset{P'}{i+16}$

claim: Every $\delta/2 \times \delta/2$ ~~box~~ has at most one point from S.

Pf (idea) claim: by contradiction.

Assume a $\delta/2 \times \delta/2$ square has points p & q.



p & q are farthest apart when on the opposite ends of a diagonal.

$$d(p,q) = \sqrt{(\delta/2)^2 + (\delta/2)^2}$$

$$= \sqrt{\frac{2\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$$

$\Rightarrow$ contradicts the defn of δ: $\min(d(q_0, q_1), d(r_0, r_1))$

(as every $\delta/2 \times \delta/2$ square is entirely in Q or is in R)