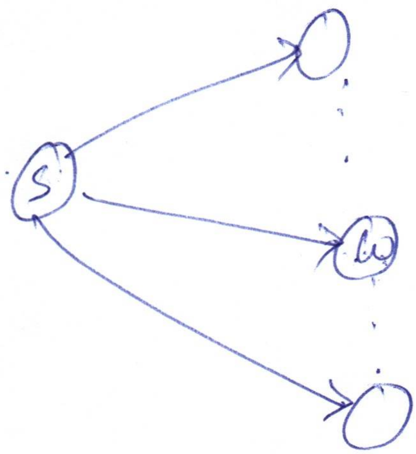
Attempt 3: $OPT(s)$ = cost of shortest s-t path $\forall s \in V$.

(•) How many subproblems?

$OPT(s)$? n ✓

(•) Recurrence / recursive defn of subproblems:



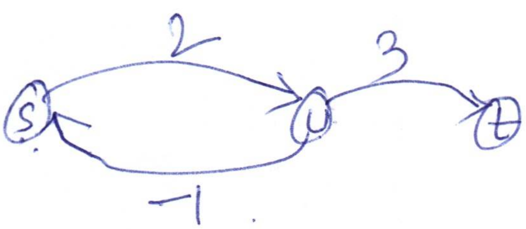
if (s,w) in in shortest s-t path:

$$OPT(s) = c_{s,w} + OPT(w)$$

In general,

$$OPT(s) = \min_{w:(s,w) \in E} \{c_{s,w} + OPT(w)\}$$

(•)



$$OPT(s) = 2 + OPT(u)$$

$$OPT(u) = \min\{3 + OPT(t), -1 + OPT(s)\}$$

ISSUE: $OPT(s)$ depends on $OPT(u)$
$OPT(u)$ " " $OPT(s)$

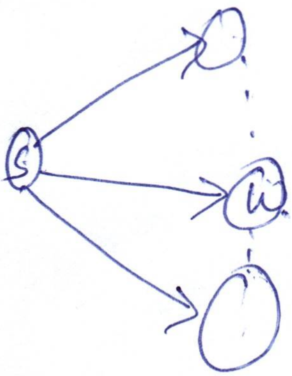
Apr 29

Intuition: If we use edge (s,w) when we compute shortest s-t path, we cannot reuse the same edge when computing shortest u-t path.

idea/solution: Introduce an implicit parameter to define the subproblem.

Attempt 4: * $OPT(s, E')$: cost of the shortest path using edges in E'; $(E' \subseteq E)$.

$$OPT(s, E') = \min_{\substack{w: \\ (s,w) \in E}} \{ C_{s,w} + OPT(w, E' \setminus \{(s,w)\}) \}$$



* ordering among subproblems: order the subproblems according to the size of E' (i.e., $|E'|$)

* How many subproblems: $OPT(s, E')$ $\forall s \in V$ $\quad$ $n \cdot 2^m$

Set: $\{1, 2, \ldots, n\}$ How many subsets? 2^n

Note: (1) we do not need to keep track of all edges that we have not used yet.

(2) Keep track of how close we are to t.

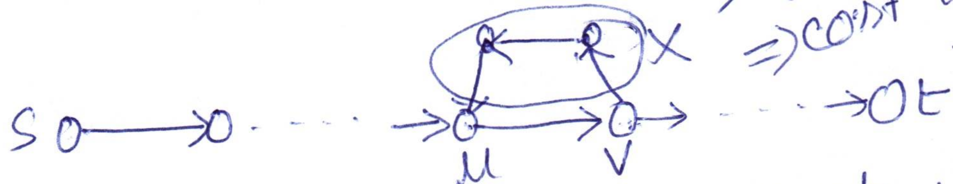
Bellman-Ford

Attempt 5: * $OPT(s, i)$ = cost of the shortest s-t path using only $\leq i$ edges.

Proposition: If G has no negative cycles, then $\exists$ a shortest s-t path that is simple.

$\Rightarrow$ has $n-1$ edges.

(Pf idea):



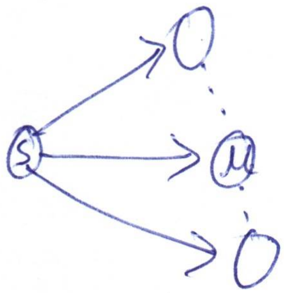
$\Rightarrow$ fewer edges

$\Rightarrow$ cost is no greater than before

$\rightarrow$ remove repeat visits to u and v

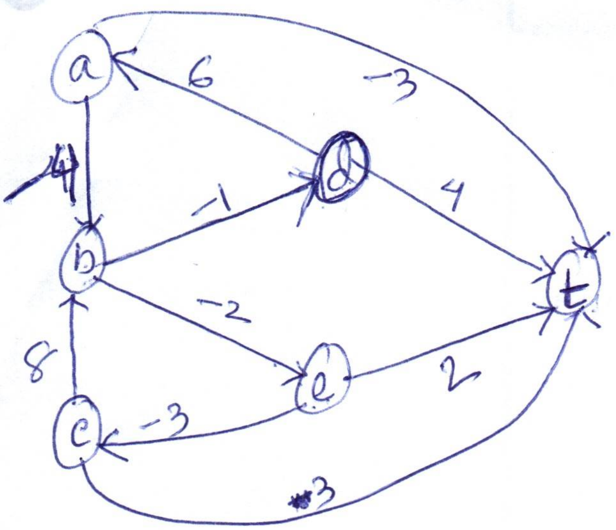
$$OPT(s,i) = c_{s,u} + OPT(u, i-1)$$

In general,



$$OPT(s,i) = \min_{u:\,(s,u)\in E} \{ c_{s,u} + OPT(u, i-1) \}$$

$\Rightarrow$ Goal: output $OPT(s, n-1)$ $\forall s \in V$



focus on vertex d

$OPT(d,0) = \infty$ [as $d \neq t$]

$OPT(d,1) = 4$ [d, t]

$OPT(d,2) = 6-3=3$ [d, a, t]

$OPT(d,3) = 3$ [d, a, t]

$OPT(d,4) = \underline{6+4-2+2}$ ✗ [d, a, b, e, t]

$= 6-4-2+2 = 2$

$OPT(d,5) = 6-4-2-3+3 = 0$ [d, a, b, e, c, t]

$OPT(d,6) = OPT(d,5) = \cdots = 0$ $\boxed{n-1=5}$

$= OPT(d,7) = OPT(d,8) = \cdots$

$OPT(t,0) = 0$ $OPT(u,0) = \infty \quad \forall u \neq t$

$OPT(u,i)$ for $i > 0$