Feb 11

Gale-Shapley Algorithm

⓪ Initially all men/women are free

① # In a loop, (in textbook, men propose) a free woman proposes to a man.

② We have n matched pairs.

Example

n=2; M={BP, BBT}; W={JA, AJ}

L_{JA}: BP > BBT	L_{BP}: AJ > JA
L_{AJ}: BBT > BP	L_{BBT}: JA > AJ

F: free, E: engaged

AJ	JA	BP	BBT
F	F	F	F

⓪ Initial state: all n men and n women are free.

① Let w be a free woman
Q1: which man m should w propose to?
A1: the man m on top of L_w
Q2: what should m do?
A2: Accept? Reject? ✗ (m, w) get engaged

(AJ → BBT) proposal made
what should BBT do?
Accept?
Reject? ✗ (BBT, AJ) get engaged

AJ	JA	BP	BBT
E	F	F	E

Obs All n men & women are either engaged or some are free.

① All n men and n women are engaged.
Q: what would the algm. do?
A: The algm. terminates and outputs n matched pairs.

② ∃ a ~~woman~~ free woman w

(Q1) ~~who~~ who should w propose to?

(A1) The best man m that w has not proposed to yet.

(Q2) what should m do?

Case 1: m is free
⇒ (m, w) get engaged.

Case 2: m is engaged to w'. [w ≠ w']

Case 2.1: w' > w in Lm.
⇒ nothing changes.
w is free.

Case 2.2: w > w' in Lm.
(m, w) get engaged.
w' is free.

AJ	JA	BP	BBT
E	F	F	E

Q1: who should JA propose to? (JA → BP) proposal made.

Q2: what should BP do?
~~(JA,~~ (BP, JA) get engaged.

AJ	JA	BP	BBT
E	E	E	E