Feb 14

THEOREM: For any input (M, W, 2n pref. lists) the GS alg. outputs a stable matching.

|M| = |W| = n

Corollary: Every input to a stable matching problem has a stable matching. → [Run GS on the input → stable matching]

Proof of THEOREM:

let S be the output of GS alg. on an arbitrary input. → [you want to prove that S is a stable matching.

└ Q: How?

└ Q1: What makes a matching stable?

Q1.2: Does S meet the requirements for stability?

{ 1. S is a perfect matching
{ 2. S has no instability

Lemma 1: The GS alg. terminates $\leq n^2$ iterations.

Lemma 2: S is a perfect matching.

Lemma 3: S has no instability.

Lemma 1 + Lemma 2 + Lemma 3 $\Longrightarrow$ THEOREM

Proof idea of lemma 1:

In each iteration a new proposal is made from $w \to m$ $w \in W$, $m \in M$.

$$\#\text{iterations} = \#\text{proposals} \leq \text{pair}(m, w) = n^2$$

$m \in M$
$w \in W$

Cartesian product

$$= |M \times W| = |M| \times |W| = n \times n = n^2$$

$$\#\text{iterations} \leq n^2$$