Feb 16

Proof of Lemma 2 (S is a perfect matching)

Obs 0 : S is a matching ($\Rightarrow$ the output S $\Rightarrow$ n engaged pairs) $|M| = |W| = n$

Obs 1 : Once a man gets engaged, he keeps getting engaged with better women.

Obs 2 : If w proposes to m after m', m' > m in L_w

Lemma 4 : If at the end of an iteration w is free, w has NOT proposed to all men.

Proof idea : proof by contradiction (use obs 0 + lemmas 2, 4, alg. def.)

Proof details : Assume S is not a perfect matching.

$\xRightarrow{\text{obs 0}}$ $\exists$ a free woman - w.

$\xRightarrow{\text{by lemma}}$ $\exists$ a man m that w has not proposed to $\longrightarrow$ (*)

By lemma 1, GS terminates $\Longrightarrow$ ~~a free~~ all women proposed to all men.

$\Longrightarrow$ Contradicts (*)

Pigeonhole Principle (PHP): $\exists \le n-1$ pigeons and n pigeonholes $\Longrightarrow$ ~~at le~~ ≥ 1 empty pigeonholes.

proof details:

Assume w is free and w has proposed to all men.

$\xRightarrow[\text{Obs 1}]{}$ all men are engaged $\longrightarrow$ (*)

(By ~~Lem~~ Obs 1)

Since w is free $\Longrightarrow$ $\le n-1$ women are engaged.

$\xRightarrow[\text{PHP}]{}$ ≥ 1 man is not engaged.

PHP
pigeonholes: m
Pigeons: w
assignment: engaged

$\Longrightarrow$ Not all men are engaged $\longrightarrow$ (*)