Graphs

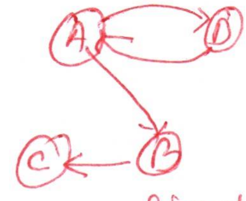
$G = (V, E) \qquad E \subseteq V \times V$

set of vertices   set of edges

UnDirected

$V = \{A, B, C\}$

$E = \{(A,B), (B,C), (C,A)\}, (B,A), (C,B), (A,C)\}$



directed

$V = \{A, B, C, D\}$

$E = \{(A,B), (B,C), (A,D), (D,A)\}$

$n = 4$

$m = 4$

Defaults: $|V| = n$;

$|E| = m$

Def: G is undirected $\iff \forall\, u \neq w \in V,\ (u,w) \in E \iff (w,u) \in E$

u •——• v $\equiv$ 

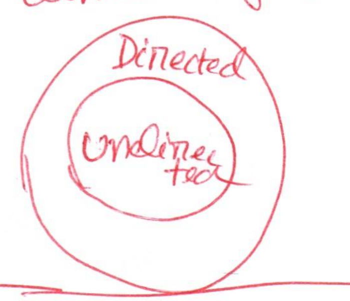
Q: (·) Airline map (undirected) (·) Wikipedia articles map (directed)

Default: G is undirected.

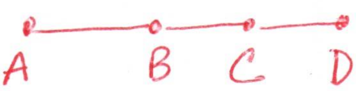
Claim: Every undirected graph is also a directed graph

Pf. idea: replace every 

$\Rightarrow$ 

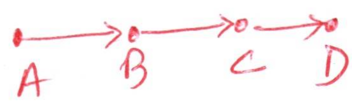


Paths:



is path?

D, C, B, A ✓	3
A, B, C, D ✓	3
A, B, C, B ✓	3
A, C, D ✗	—



D, C, B, A ✗

A, B, C, D ✓

A, B, C, B ✗

A, C, D ✗

Def: A path in (directed) $G = (V, E)$ is a sequence of vertices $u_1, \ldots, u_K$ $\{u_1 - u_K \text{ path}\}$ s.t. $\forall\, i \in [K-1] = \{1, \ldots, K-1\}$ $(u_i, u_{i+1}) \in E$

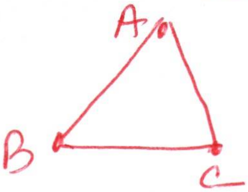
Notes: (i) u_i need not be distinct (ii) holds for directed G

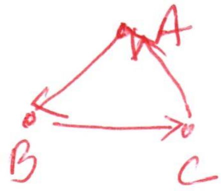
Def: A simple path is a path w/ no repeated vertices.

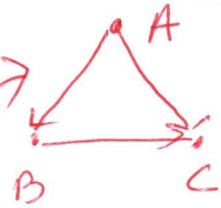
Ex: Any simple path has length $\leq n-1$

Def: the length of a path is the number of edges in it.

Cycles:







cycle? : A, C, B, A ✓
A, B, C, A ✓

A, C, B, A ✗
A, B, C, A ✓

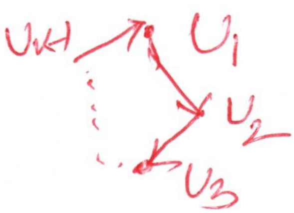
A, C, B, A ✗
A, B, C, A ✗

Directed acyclic graph (DAG)

Def: A cycle is a sequence

$v_1, v_2, \ldots, v_k = v_1$ $\quad v_1, \ldots, v_{k-1}$ are distinct

$\forall i \in [k-1], \quad (v_i, v_{i+1}) \in E$



How small can k be?
Undirected: 3 Directed: 2 (?)