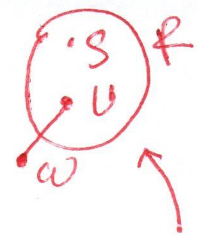
Explore (s)



0. $R \leftarrow \{s\}$
1. While $\exists\, (u,w) \in E$ s.t. $w \notin R$, $u \in R$
   add w to R
2. Output $R^* \leftarrow R$

<u>THEOREM</u>: For all G, start vertices s, $R^* = cc(s)$.

<u>Corollary</u>: BFS is correct.

<u>General Idea</u>: To show that two set A and B are equal, i.e., $A = B$, show
i) $A \subseteq B$ AND ii) $B \subseteq A$

<u>Lemma 1</u>: $R^* \subseteq cc(s)$ $\Leftarrow$ Everything that is output by Explore is correct.

<u>Lemma 2</u>: $cc(s) \subseteq R^*$ $\Leftarrow$ Everything that should be output, ~~by Expl~~ is output by Explore.

<u>Proof of Lemma 1</u>: by Induction.

Proof idea of lemma 2 ($cc(s) \subseteq R^*$)

Note: Since R^* is output by Explore $\Rightarrow$ Explore has terminated (*)

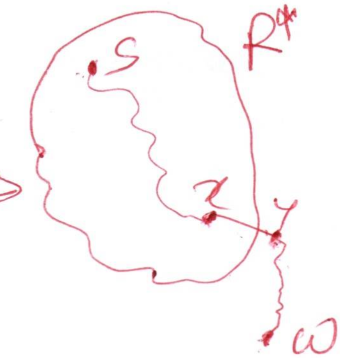
Proof by Contradiction:

$\{1,2,3\} \not\subseteq \{1,2\}$

Assume $cc(s) \not\subseteq R^*$

$\Rightarrow \exists$ a vertex w s.t. $w \in cc(s)$, $w \notin R^*$

$\Rightarrow \exists$ $s-w$ path P in G, but $w \notin R^*$



Since P starts inside of R^* but ends up outside of R^*

$\Rightarrow$ P has to cross the boundary of R^*

$\Rightarrow \exists$ (x,y) edge s.t. $x \in R^*$, $y \notin R^*$

$\Rightarrow$ y should have been added to R^* by Explore

$\Rightarrow$ Explore could not have been terminated

$\Rightarrow$ Contradict (*) $\square$