Lemma 1: At the end of each iteration, if $u \in R$, the path P_u is a shortest s-u path.

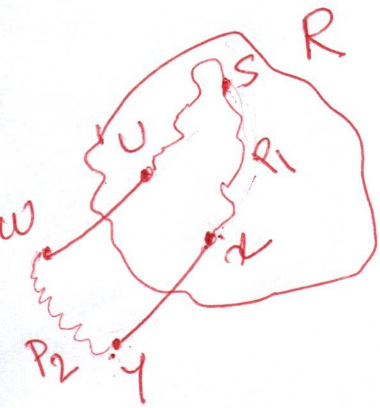
Proof (idea): By induction on $|R|$ ($\#$ vertices $= \#$ iteration)

Base case: $|R| = 1$, $R = \{s\}$, $d(s) = 0$

I. H.: Assume that lemma 1 is true for $|R| = k$, $\forall k \geq 1$!

I. S.: $|R| = k+1$. Assume w has been added to R as the $(k+1)$th vertex.

Goal: P_w is the shortest s-w path.



* Assume vertex $U \in R$ discovered w.

$d(w) = d(U) + l_{U,w}$

$P_w = P_{U,w}$

→ Proof (idea): By contradiction.

Assume $\exists$ an s-w path $P'_w \neq P_w$ s.t. $l(P'_w) < l(P_w)$ (*)

As $s \in R$, but $w \notin R$, P'_w "crosses" the boundary of R at some edge (x, y)

$P'_w = P_1, x, y, P_2$

$$l(P'_w) = l(P_1) + l(x,y) + l(P_2) \geq d(x) + l(x,y) + l(P_2)$$
$$\geq d'(y) + l(P_2)$$
$$\geq d'(y) \quad [l(P_2) \geq 0]$$
$$\geq d'(w) \text{ as algo chose } w \text{ every}$$
$$= d(w) = l(P_w)$$

$l(P'_w) \geq l(P_w)$
$=$ contradicts (*) $\square$