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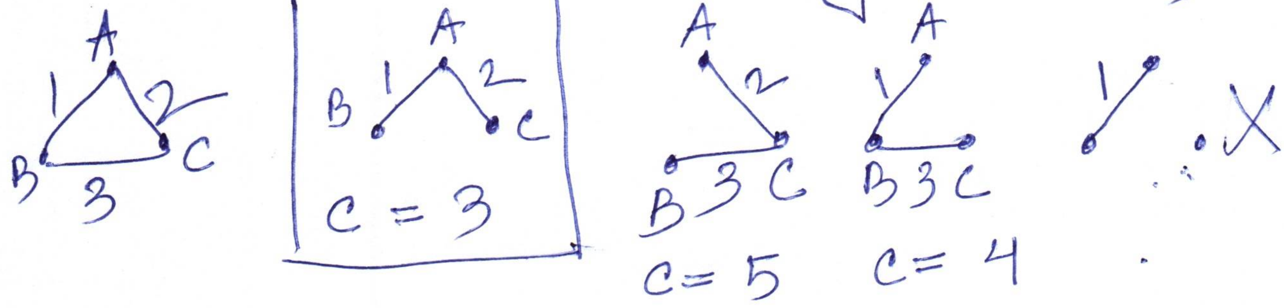
Minimum Spanning Tree (MST)



Input: A connected, undirected graph $G = (V, E)$, $C_e \geq 0 \ \forall e \in A$

C_e : cost of edge e

Output: $E' \subseteq E$ s.t.

① $T = (V, E')$ is connected.

② $C(T) = \sum_{e \in E'} C_e$ is minimized.

Proposition: Let $C_e > 0 \ \forall e \in E$

Any optimal solution $T = (V, E')$ is a tree.

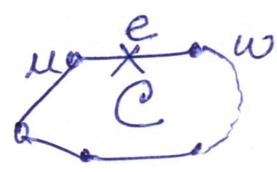
(*)

proof (idea): By contradiction.

~~T is not a~~ Assume ~~that~~ T is not a tree.

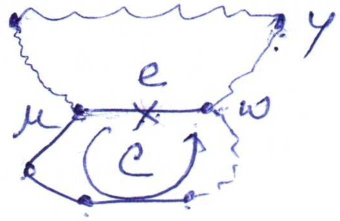
$\Rightarrow \exists$ a cycle C in T.

Let e be an edge on C.



Remove e from $\underset{T}{\underline{C}}$ to get $T' = (V, E' - \{e\})$

claim 1: $c(T') < c(T)$



$c(T') = c(T) - c_e$

$< c(T)$, as $c_e > 0$

$T' = (V, E' - \{e\})$

claim 2: T' is still connected.

Case 1: $\exists$ an x-y path that doesn't use edge e.

$\Rightarrow$ x-y is still connected

Case 2: all x-y paths use edge e.

$\Rightarrow$ Use the rest of C to connect u and w. (i.e., takes the longer route)

$\Rightarrow$ x-y path is still existing

$\Rightarrow$ x, y are connected

claim 1 + claim 2 $\Rightarrow$ T' is a better solution

$\Rightarrow$ T can't be optimal.

$\Rightarrow$ contradicts (*)