May 11

$\Rightarrow \quad P \stackrel{?}{=} NP \quad \Rightarrow \quad P \subseteq NP$ AND $NP \subseteq P$

$\Rightarrow \quad P \subseteq NP$

Pf: $\forall Y, Y \in P \Rightarrow \exists$ an algo A s.t. $A(w) = 1 \Leftrightarrow w \in Y$

verifier (B): $B(w, t) = A(w)$

Lemma 1: Let X be NP-complete.

$X \in P \Leftrightarrow P = NP$

Pf (idea): $\Longrightarrow$

X in NPC.

$X \in P \Rightarrow \forall Y \in NP, Y \leq_p X$

$\Rightarrow Y \in P \Rightarrow NP \subseteq P$

$\Rightarrow P = NP$

$\Leftarrow$ X in NPC

$P = NP \Rightarrow X \in NP$

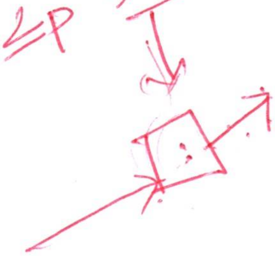
$\Rightarrow X \in P$

$\square$

Algo SAT (φ)

1. Convert φ to G_φ
2. $b \leftarrow$ Algo-3-color(G_φ)
3. Return b.

3-SAT $\leq_P$ 3-colorings



Step 1: Construct a graph G_0 on $2n$ vertices:

$V_1, \dots, V_n$ with $x_i \equiv V_i$

$\overline{V_1}, \dots, \overline{V_n}$ $\quad \overline{x_i} \equiv \overline{V_i}$

and 3 special nodes T, F, B.

if $c(V_1) = c(T)$
$\Rightarrow c(\overline{V_1}) = c(F)$

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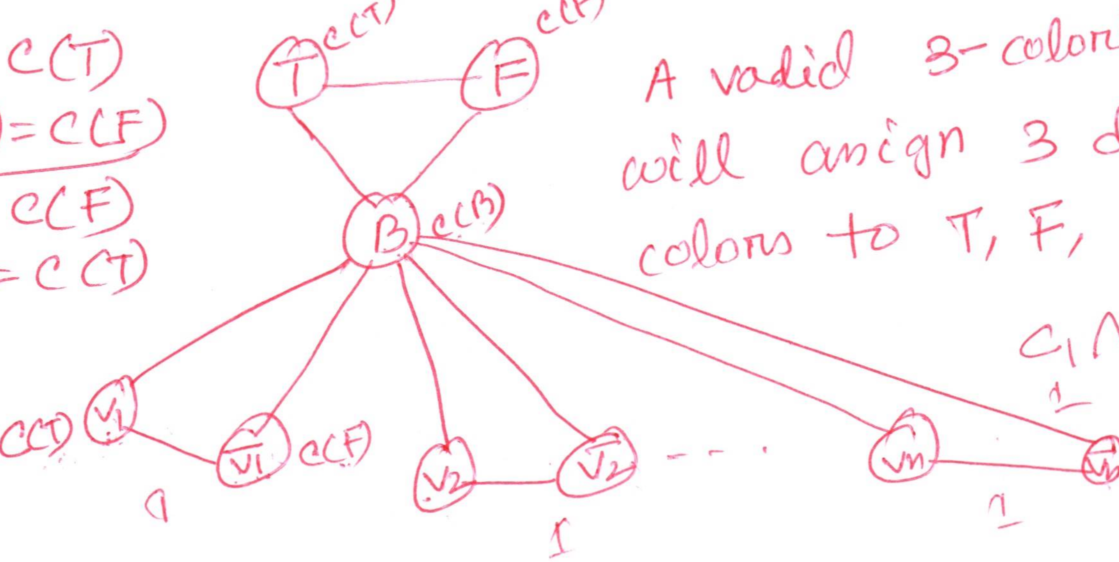
A valid 3-coloring will assign 3 distinct colors to T, F, B.

$C_1 \wedge C_2 \wedge C_3 \wedge \dots \wedge C_m$

G_0:



$\exists$ a valid 3-coloring of $G_0 \Leftrightarrow \exists$ an assignment of $x_1, \dots, x_n$ s.t.

if $c(V_i) = c(T)$
$x_i = 1$
else $x_i = 0$.