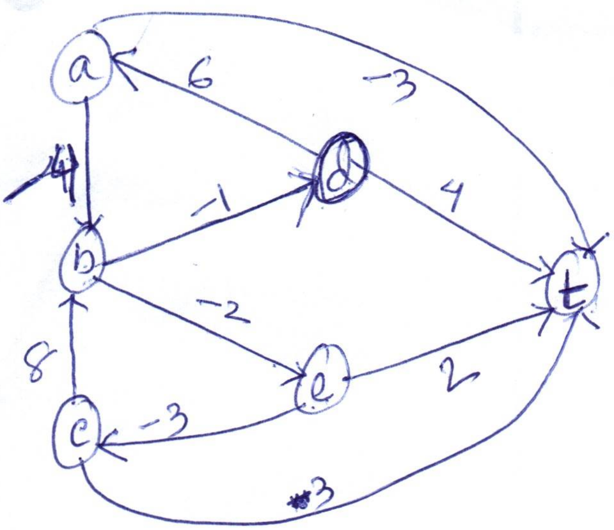


focus on vertex d

$OPT(d,0) = \infty$ [as ~~d~~ $d \neq t$]

$OPT(d,1) = 4$ [d, t]

$OPT(d,2) = 6-3 = 3$ [d, a, t]

$OPT(d,3) = 3$ [d, a, t]

$OPT(d,4) = \underline{6 + 4 - 2 \cancel{+2}}$ ✗ [d, a, b, e, t]

$= 6-4-2+2 = 2$

$OPT(d,5) = 6-4-2-3+3 = 0$ [d, a, b, e, c, t]

$OPT(d,6) = OPT(d,5) = \dots = 0$ $\quad \boxed{n-1=5}$

$= OPT(d,7) = OPT(d,8) = \dots$

$OPT(t,0) = 0 \qquad OPT(u,0) = \infty \quad \forall u \neq t$

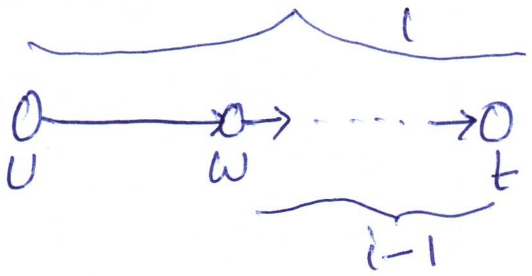
* $OPT(u,i)$ for $i > 0$

$\boxed{\text{May 2}}$

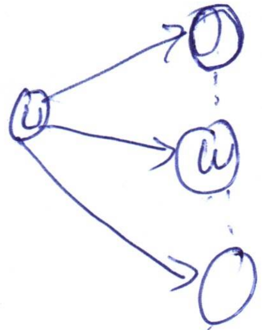
Case 1: $\exists$ a shortest u-t path using $\leq i-1$ edges.

$OPT(u,i) = OPT(u,i-1)$

<u>Case 2:</u> Shortest v–t paths use exactly i edges



$$OPT(v,i) = c_{v,w} + OPT(w,i-1)$$



In general,

$$OPT(v,i) = \min_{\substack{w:\\(v,w)\in E}} \{ c_{v,w} + OPT(w,i-1) \}$$

<u>Overall:</u> ← Recurrence

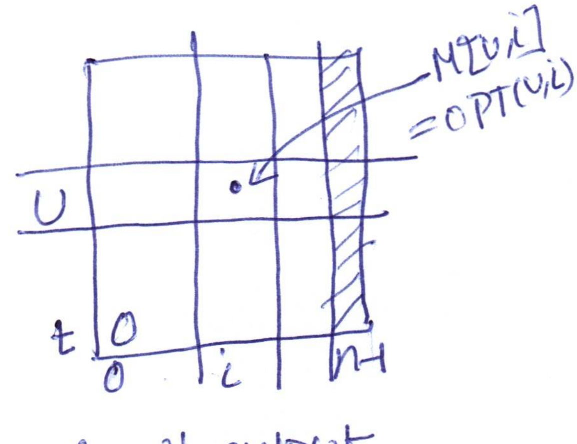
$$OPT(v,i) = \min\Big\{ OPT(v,i-1),\ \min_{\substack{w:\\(v,w)\in E}} \{ c_{v,w} + OPT(w,i-1) \} \Big\}$$

* How many subproblems?: $OPT(s,i)$ $\forall s \in V$, $0 \le i \le n-1$ — $n \cdot n = O(n^2)$

* Ordering among subproblems:

Column i depends on column $i-1$.

Construct the matrix M column by column → $L \to R$



Goal: output $M[s, n-1]$ $\forall s$

Bellman-ford alg.

0. Allocate an $n \times n$ matrix $\} O(n^2)$

1. $M[t,0] \leftarrow 0$, $M[v,0] \leftarrow \infty \quad \forall v \neq t$

   $O(n)$

2. for $i = 1 \cdots n-1$

   $O(n^3)$ for $v \in V$

   $O(n) \; M[v,i] \leftarrow \min\left\{ M[v,i-1], \min_{(v,w) \in E} \{ c_{v,w} + M[w,i-1] \} \right\}$

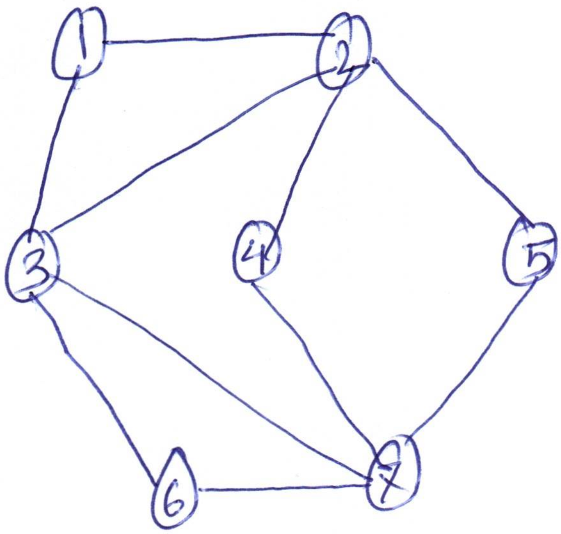
3. Return $M[s, n-1] \quad \forall s \in V$

   $O(n^3)$

→ Improvement in Runtime analysis

$\Rightarrow O(mn)$ $\qquad \sum_{v \in V} \deg(v) = m$

$G = (V, E)$



Independent set (IS) problem

Defn: An IS is a subset $S \subseteq V$ if $\exists$ no ~~end~~ edges between any two vertices in S

$\{1, 4\}$ ✓ $\{3, 4, 5\}$ ✓

$\{3, 7\}$ ✗ $\{1, 4, 7, 5\}$ ✗

$\{1, 4, 5, 6\}$ ✓