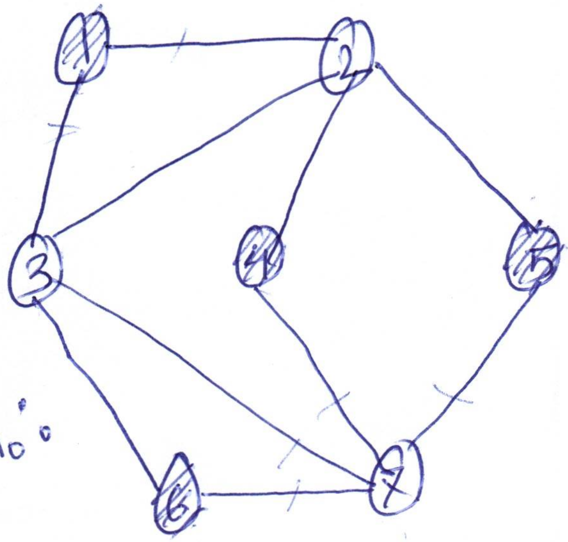
Problem 1

$G = (V, E)$



G_0:

Independent set (IS) problem

Defn: An IS is a subset $S \subseteq V$ if $\exists$ no ~~end~~ edges between any two vertices in S

$\{1,4\}$ ✓ $\{3,4,5\}$ ✓

$\{3,7\}$ ✗ $\{1,4,7,5\}$ ✗

S $\{1,4,5,6\}$ ✓

May 4

Input: $G = (V, E)$; $0 \le K \le n$ $(|V| = n)$

Output: TRUE if $\exists$ an IS of size $\geq K$

Eg: G_0; 2 ✓, G_0; 3, G_0; 4 ✓, G_0; 5

$\{1,4\}$ $\{3,4,5\}$ ✓ $\{1,4,5,6\}$

Problem 2: Vertex cover (VC) problem

A subset $C \subseteq V$ is a VC if every edge e has at least one end point in C.

G_0 $\{1,2,3,4,5,6,7\}$ ✓ $\{1,2,3,4,5,6\}$ ✓

$\{1,2,6,7\}$ ✓ $\{2,3,7\}$ ✓ $V-S$ $\{1,7\}$ ✗

Note: Any subset of size $n-1$ is a VC.

Input: $G = (V, E)$; $0 \leq k \leq n$

Output: TRUE if $\exists$ a VC of size $\leq k$.

Eg: G_0; 6 ✓ G_0; 3 ✓ G_0; 2 ✗

THM (1) $IS \leq_p VC$

(2) $VC \leq_p IS$

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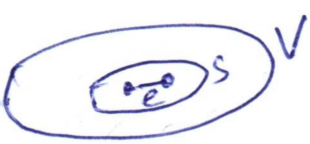
lemma: $G = (V, E)$

$S \subseteq V$ is an IS $\iff$ $V \setminus S$ is a VC

Pf (idea): $\Longrightarrow$ let S be an IS (*)

for contradiction, assume that $V \setminus S$ is not a VC.



$\Rightarrow$ $\exists$ an edge e that has no end points in $V \setminus S$.

$\Rightarrow$ e is completely inside S.

$\Rightarrow$ contradicts (*)

$\Leftarrow$ let $V \setminus S$ be a VC. (*)

for contradiction, assume S is not an IS

$\Rightarrow$ $\cancel{\exists}$ $\exists$ an edge e that has both endpoints in S.

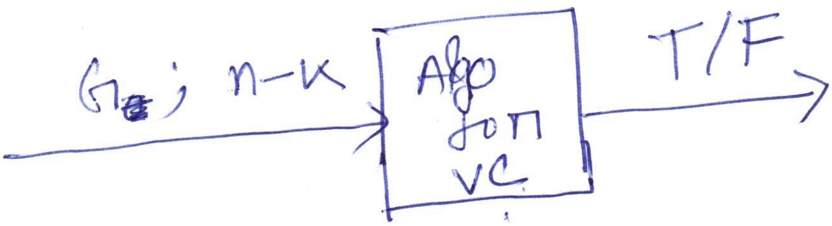
$\Rightarrow$ contradicts (*)

$\square$

Corollary: G has an IS of size $\geq K$

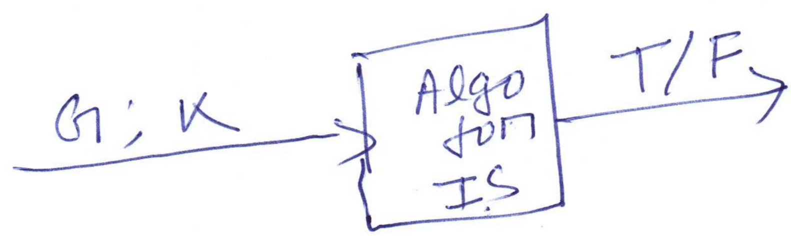
$\iff G$ " a VC " " $\leq n-K$.

THM (I) $\underset{Y}{IS} \leq_p \underset{X}{VC}$ $\quad$ $\leq_p$: poly-time reducible



T/F for IS as well.

THM (II)



T/F for VC.

Satisfiability / SAT problem

SAT formula: Conjunction / AND of clauses.
↳ OR/disjunction of literals
↳ $\{x_i, \bar{x}_i\}$

Eg: $(x_1 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee \bar{x}_3) \wedge (x_2 \vee \bar{x}_3)$

OR AND

In general: $\left.\begin{array}{l} C_1 \wedge C_2 \wedge \cdots \wedge C_m \\ \equiv C_1, C_2, \cdots, C_m \end{array}\right\}$ m clauses

A set of variables/literals $X : \{x_1, x_2, \cdots, x_n\}$

clauses: disjunction/OR of literals: $t_1 \vee t_2 \vee \cdots \vee t_\ell$

$$t_i \in \{x_1, x_2, \ldots, x_n, \overline{x_1}, \overline{x_2}, \ldots, \overline{x_n}\}$$

Assignment: $v : X \to \{0,1\}$

$n = 3$

$x_1 =$	0	1	0
$x_2 =$	0	1	0
$x_3 =$	0	1	1

How many assignments? 2^n

$$\frac{x_1}{0/1}\ \frac{x_2}{0/1}\ \cdots\ \frac{x_n}{0/1} \leftarrow \text{2 choices for each } x_i$$

A satisfying assignment to a SAT formula φ ~~is~~ is an assignment on which φ evaluates to true.