clauses: disjunction/OR of literals: $t_1 \vee t_2 \vee \cdots \vee t_l$

$$t_i \in \{x_1, x_2, \ldots, x_n, \overline{x_1}, \overline{x_2}, \ldots, \overline{x_n}\}$$

Assignment: $v: X \to \{0,1\}$

$n = 3$

$x_1 = 0$	1	0
$x_2 = 0$	1	0
$x_3 = 0$	1	1

How many assignments? 2^n

$\frac{x_1}{0/1}$ $\frac{x_2}{0/1}$ $\cdots$ $\frac{x_n}{0/1}$ $\leftarrow$ 2 choices for each x_i

A satisfying assignment to a SAT formula φ is an assignment on which φ evaluates to true.

May 16

Eg: $(0,0,0)$ — x_1, x_2, x_3

$x_1 \vee \overline{x_2} = 0 \vee \overline{0} = 0 \vee 1 = 1$
$\overline{x_1} \vee \overline{x_3} = \overline{0} \vee \overline{0} = 1 \vee 1 = 1$
$x_2 \vee \overline{x_3} = 0 \vee \overline{0} = 0 \vee 1 = 1$

$1 \wedge 1 \wedge 1 = 1$
$(0,0,0)$ is a satisfying assignment for φ.

$(1,1,1)$ — x_1, x_2, x_3

$x_1 \vee \overline{x_2} = 1 \vee \overline{1} = 1 \vee 0 = 1$
$\overline{x_1} \vee \overline{x_3} = \overline{1} \vee \overline{1} = 0 \vee 0 = 0$
$x_2 \vee \overline{x_3} = 1 \vee \overline{1} = 1 \vee 0 = 1$

$1 \wedge 0 \wedge 1 = 0$
$(1,1,1)$ is NOT a satisfying assignment φ.

Q: Given a SAT formula φ, ~~does~~ $\exists$ a satisfying assignment?

$\Rightarrow$ is φ satisfiable?

3-SAT formula: A SAT formula $\varphi = c_1, \dots, c_m$, s.t. each clause has exactly 3 literals.

3-SAT Problem:

Input: A 3-SAT formula φ.

Output: True/1 if φ is satisfiable.

Naive algo: check all 2^n possible assignments and answer yes if $\geq$ one of them satisfies φ.

$\rightarrow O(m \cdot 2^n)$

$\Rightarrow$ 3-SAT $\leq_p$ your problem.

THM: 3-SAT $\leq_p$ IS

We'll show: given a 3-SAT formula $\varphi \xrightarrow[\text{poly-time}]{\text{reduce}} G; K$

s.t. φ is satisfiable $\Leftrightarrow$ G has an IS of size $\geq K$.

IS: i/p: $G = (V, E)$; K

O/P: yes/true/1 if G has an IS of size $\geq K$

Def: IS: A set $S \subseteq V$, s.t. no edges in S.

Step I : Replace each clause by their triangles.

$C_1 = x_1 \vee x_2 \vee x_3$, $n = 4$

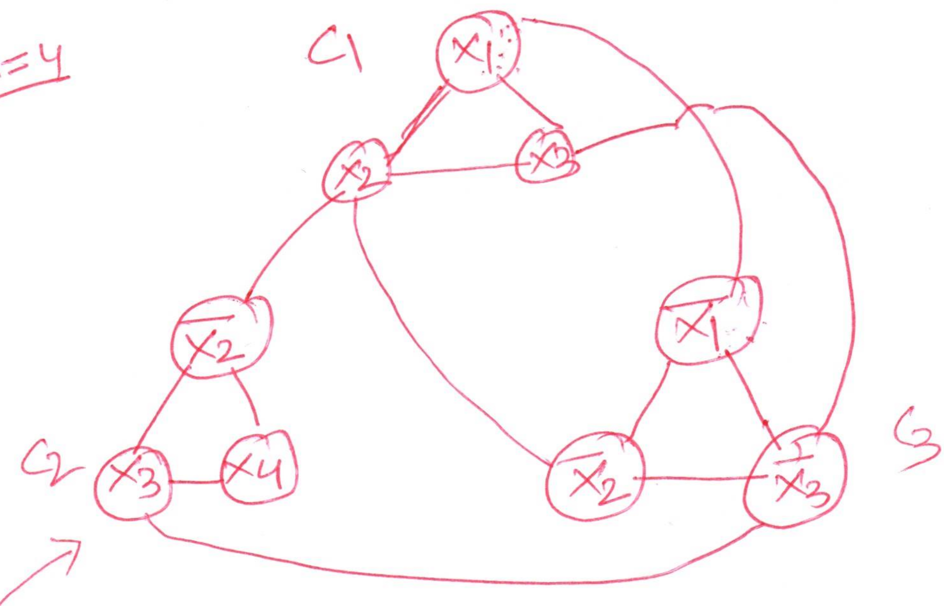
$C_2 = \overline{x_2} \vee x_3 \vee x_4$

$C_3 = \overline{x_1} \vee \overline{x_2} \vee \overline{x_3}$

$\varphi = C_1 \wedge C_2 \wedge C_3$

$1 \wedge 1 \wedge 1$

$= 1$



IS: $\{x_1, \overline{x_2}, x_4\}$

assignment : $v : \{1, 0, ?, 1\}$

φ is satisfiable $\Leftrightarrow$ G has an IS of size $\geq m$

Recall: We are considering problems Y, with output $\in \{0,1\}$

$\Rightarrow$ Y is a subset of inputs with output 1.

Defn: Q: given an input w, is $w \in Y$?

w: A 3-SAT formula

Y: All satisfiable assignments.

Def: Given an algo A & input w, $A(w) \in \{0,1\}$ denotes the output of algo A on input w.

Def: Algo A solves problem Y if $\forall$ inputs w

$$A(w) = 1 \iff w \in Y$$

A is a poly-time algo if $\forall$ inputs w, $A(w)$ is computed in $poly(|w|)$.

$poly(N) = N^c = N^{O(1)}$ for some c. $c = O(1)$

Def: P: A set of all (decision) problems that can be solved ~~in~~ by a poly-time algo.

Efficient Verification: $w \in Y$?

We need a certificate/witness t for Y.

Eg: 3-SAT, t = an assignment.

Def: An efficient verifier B for Y for $\forall$ input w.

IF:

1. B takes as input w & t; $B(w,t) \in \{0,1\}$
2. B runs in $poly(|w|)$
3. $w \in Y \iff \exists$ exist a string/witness t s.t. (i) $|t| \leq poly(|w|)$ (ii) $B(w,t) = 1$

Def: $Y \in NP$ if $\exists$ an efficient verifier B for Y s.t. $\forall$ input w

(I) $w \in Y \Rightarrow \exists$ an ~~efficient verifier B~~ s.t. $B(w,t) = 1$

(II) $w \notin Y \Rightarrow \forall$ '' ''

(I) $w \in Y \Rightarrow \exists$ a witness t s.t. $B(w,t)=1$

(II) $w \in Y \Rightarrow \forall$ '' t $B(w,t)=0$

---

claim: 3-SAT has an efficient verifier B.

$\updownarrow$

Y

w = a ~~3S~~ 3-SAT formula

t = an assignment v

Efficient verifier B:

check whether the assignment gives 1 on φ.

$\uparrow$ poly time

0/1	- - -	
x_1		x_n

$\Rightarrow$ 3-SAT $\in NP$

IS has an efficient verifier.

$\updownarrow$

Y

w: $G = (V,E)$; K

t: $S \subseteq V$ $|S| = K$

verifier: if $\forall u \neq v \in S$ $(u,v) \notin E$, return 1 $\uparrow$

IS $\in NP$

Def: X is NP-complete if

(i) X is NP

(ii) $\forall Y \in NP$, $Y \leq_p X$.