Def: X is NP-complete if

(i) X is NP

(ii) $\forall\, Y \in NP,\ Y \leq_p X$.

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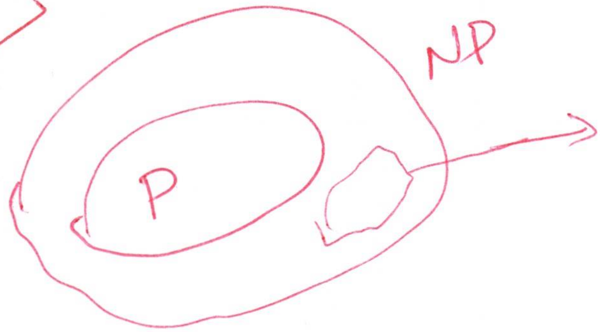

NP-complete problems are the hardest problems in NP.

⇒ THM 1: 3-SAT is NP-complete. (Book)

lemma 1: let X be NP-complete.

Then $X \in P \iff P = NP$.

lemma 2: let Y be NP-complete. $X \in NP$.

Y ↔ 3-SAT, X = IS

if $Y \leq_p X \Rightarrow X$ is NP-complete.

3-SAT $\leq_p$ IS

⇒ ~~e.g.~~

Cor 1: IS is NP-complete.

IS $\in$ NP + THM1 + 3-SAT $\leq_p$ IS + lemma 2

Cor 2: VC is NP-complete.

VC $\in$ NP + Cor1 + IS $\leq_p$ VC + lemma 2

Q: Is SP (shortest Path) problem in $\underline{P}$?

i/P: $G=(V,E)$, C_e, s, t,

O/P: a shortest s-t path?

A: Technically, NO!

Decision/Boolean version SP?

i/P: K

O/P: is there a s-t path of cost $\le K$?

Note: Notion of witness is not always obvious.

PRIMES has an efficient verifier

General Strategy to show that X is NP-complete.

Step 1: show $X \in NP$. (e.g. $X = IS$)

Step 2: Identify an NP-complete problem Y. (e.g., Y = 3-SAT)

Step 3: prove $Y \le_p X$ (e.g., 3-SAT $\le_p$ IS)