Lecture 20

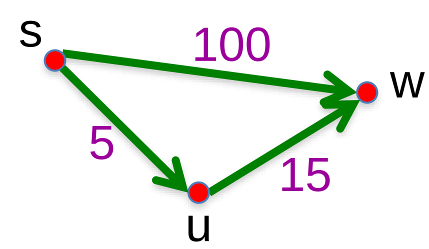
CSE 331

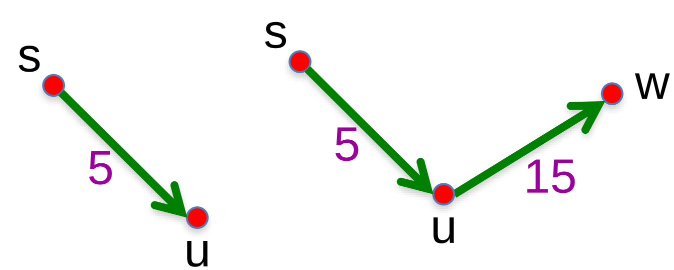
Shortest Path problem

Input: *Directed* graph G=(V,E)

Edge lengths, l_e for e in E

"start" vertex s in V



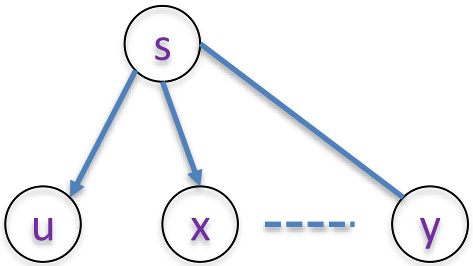


Output: Length of shortest paths from s to all nodes in V

Towards Dijkstra's algo: part one

Determine d(t) one by one

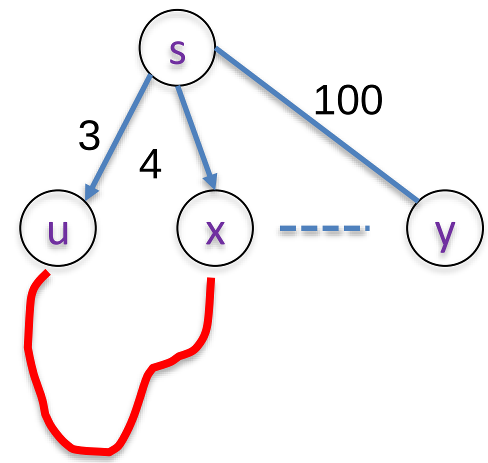
$d(s) = 0$



Towards Dijkstra's algo: part two

Determine d(t) one by one

Let u be a neighbor of s with smallest $l_{(s,u)}$



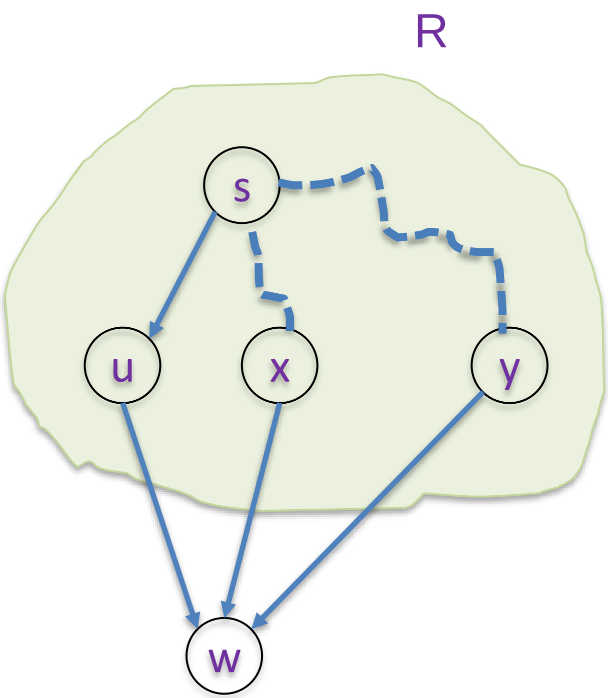
$$d(u) = l_{(s,u)}$$

Length of ~ is $\geq$ 0

Not making any claim
on other vertices

Towards Dijkstra's algo: part three

Determine d(t) one by one



Assume we know d(v) for every v in R

Compute an upper bound d'(w) for every w not in R

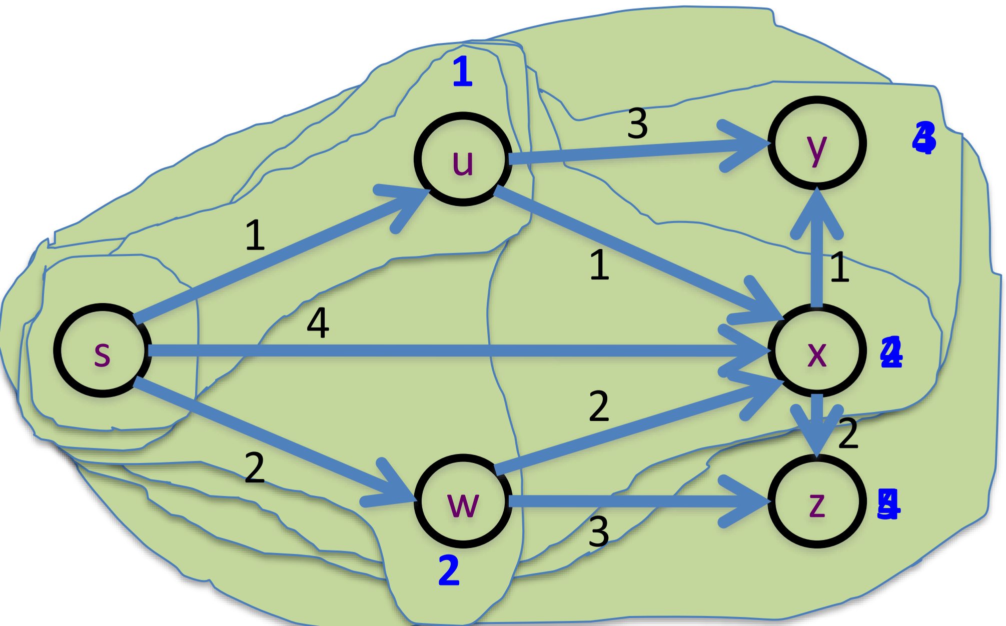
$d(w) \leq d(u) + l_{(u,w)}$

$d(w) \leq d(x) + l_{(x,w)}$

$d(w) \leq d(y) + l_{(y,w)}$

$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

Dijkstra's shortest path algorithm



$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

d(s) = 0

d(u) = 1

d(w) = 2

d(x) = 2

d(y) = 3

d(z) = 4

Input: Directed G=(V,E), $l_e \geq 0$, s in V

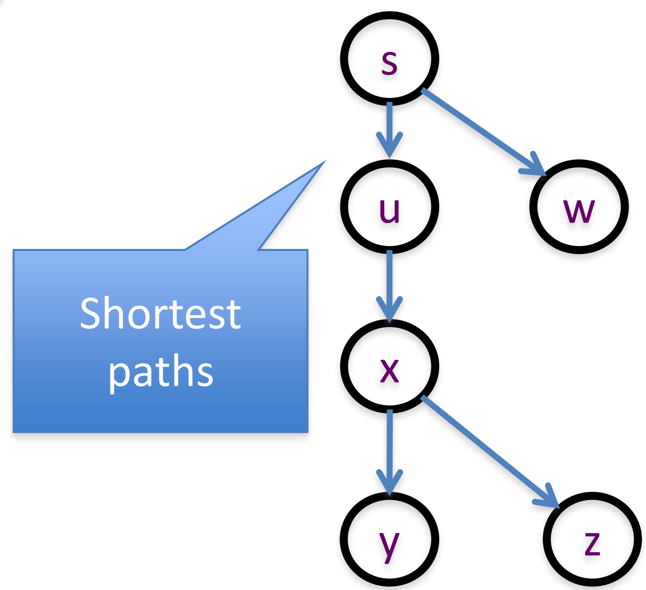
R = {s}, d(s) =0

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

Add w to R

d(w) = d'(w)



Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain "shortest path tree" separately

Dijkstra's algorithm does not work with negative weights

Left as an exercise

Rest of Today’s agenda

Prove the correctness of Dijkstra’s Algorithm

Dijkstra's shortest path algorithm

P_u shortest s-u path in "Dijkstra tree"

$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

Input: Directed $G=(V,E)$, $l_e \geq 0$, s in V

R = {s}, d(s) =0

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

Add w to R

d(w) = d'(w)

Lemma 1: At end of each iteration, if u in R, then P_u is a shortest s-u path

Lemma 2: If u is connected to s, then u in R at the end

Proof idea of Lemma 1