Lecture 23

CSE 331

Project deadlines coming up

Fri, Apr 8	Multiplying large integers F21 F19 F18 F17 x²	[KT, Sec 5.5] (**HW 5 in**) (**Project (Problem 1** `Coding` **) in**)
Week 11 Mon, Apr 11	Closest Pair of Points F21 F19 F18 F17 x²	[KT, Sec 5.4] (**Project (Problem 1** `Reflection` **) in**)

Kruskal's Algorithm

Theorem 2: Kruskal's algorithm is correct.

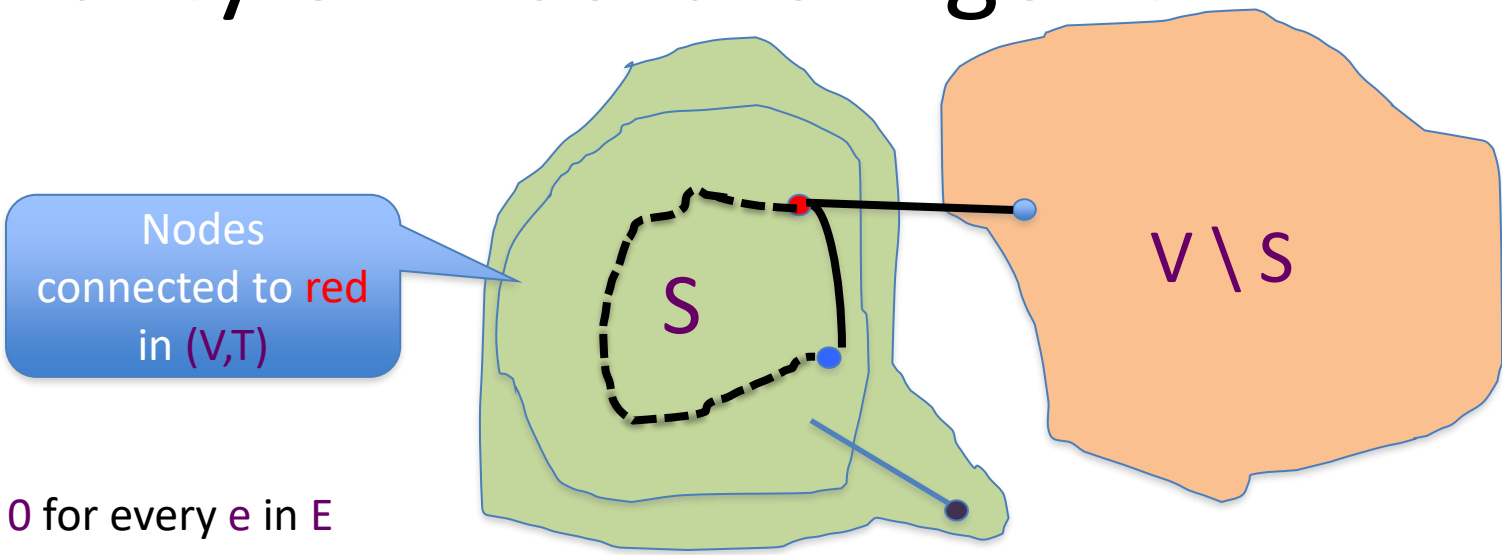
(Similar to correctness of Prim's)

Consider a run of the algorithm when it is about to add edge (u, w) to T.

Goal: show that e is the cheapest "crossing" edge across some cut (S, V\S).

Define S:
Let S be the set of vertices connected to u using only the edges in T (i.e., u has a path to all nodes in S).

Optimality of Kruskal's Algorithm



Input: G=(V,E), $c_e > 0$ for every e in E

T = Ø

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to T without adding a cycle then add it to T

S is non-empty

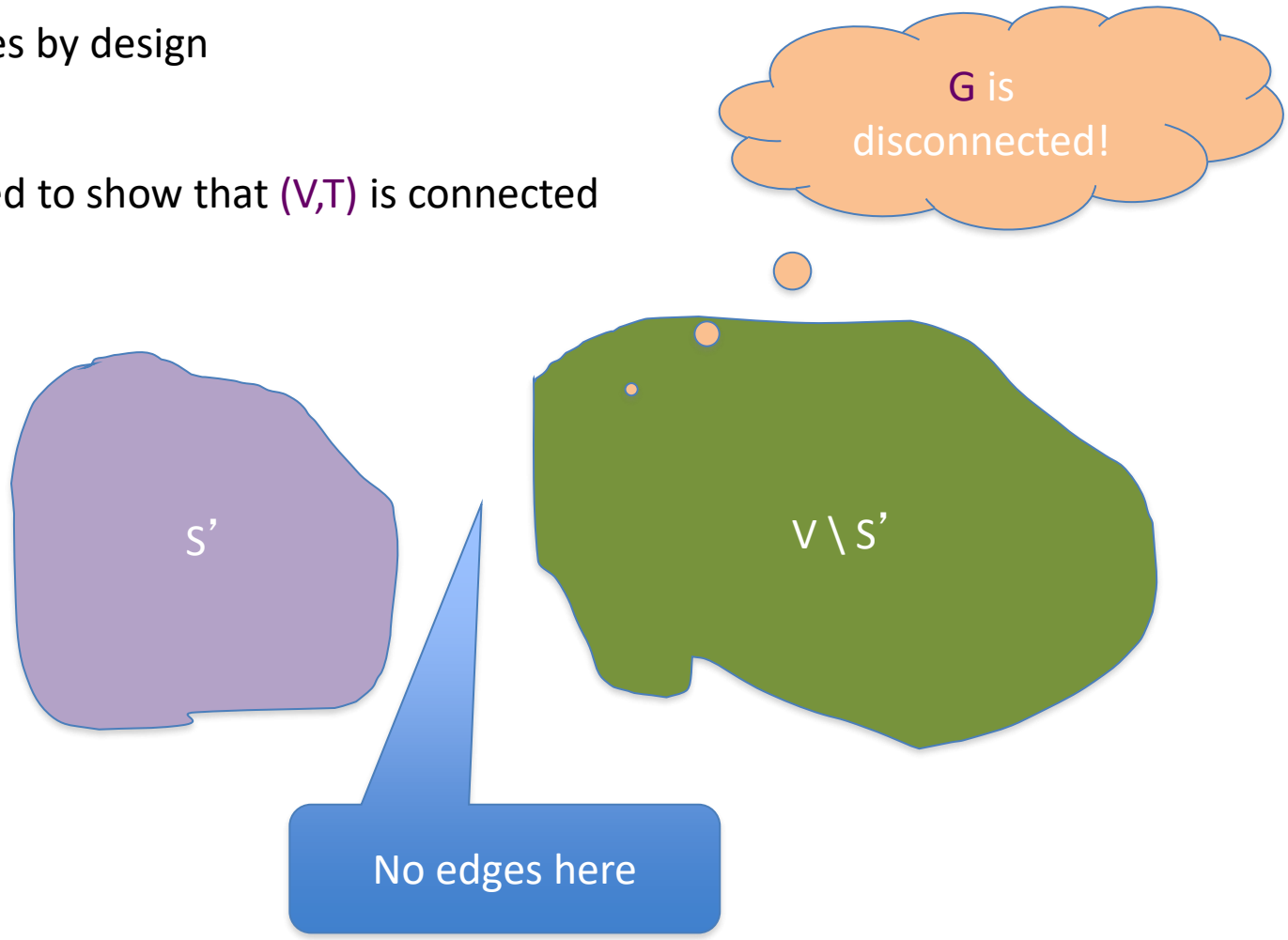
V\S is non-empty

First crossing edge considered

Is (V,T) a spanning tree?

No cycles by design

Just need to show that (V,T) is connected



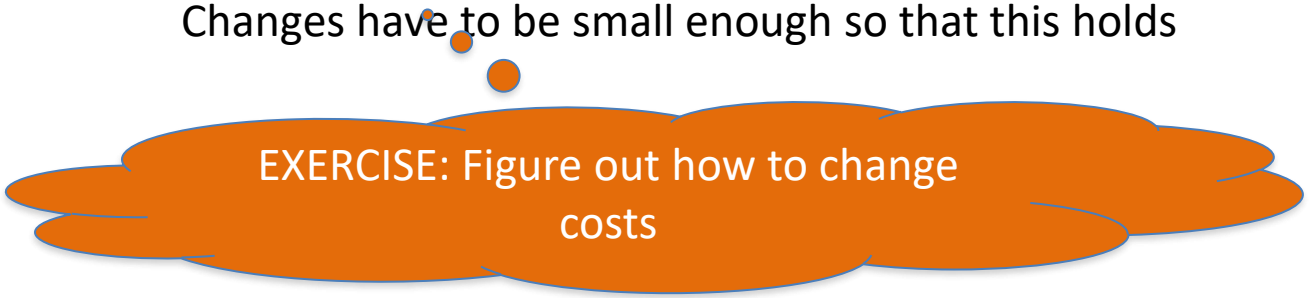
Removing distinct cost assumption

Change all edge weights by very small amounts

Make sure that all edge weights are distinct

MST for “perturbed” weights is the same as for original

Changes have to be small enough so that this holds



Running time for Prim's algorithm

Similar to Dijkstra's algorithm

O(m log n)

Input: G=(V,E), $c_e > 0$ for every e in E

S = {s}, T = Ø

While S is not the same as V

Among edges e= (u,w) with u in S and w not in S, pick one with minimum cost

Add w to S, e to T

Running time for Kruskal's Algorithm

Can be implemented in O(m log n) time (Union-find DS)

Joseph B. Kruskal

Input: G=(V,E), $c_e > 0$ for every e in E

$O(m^2)$ time overall

T = Ø

Sort edges in increasing order of their cost

Consider edges in sorted order

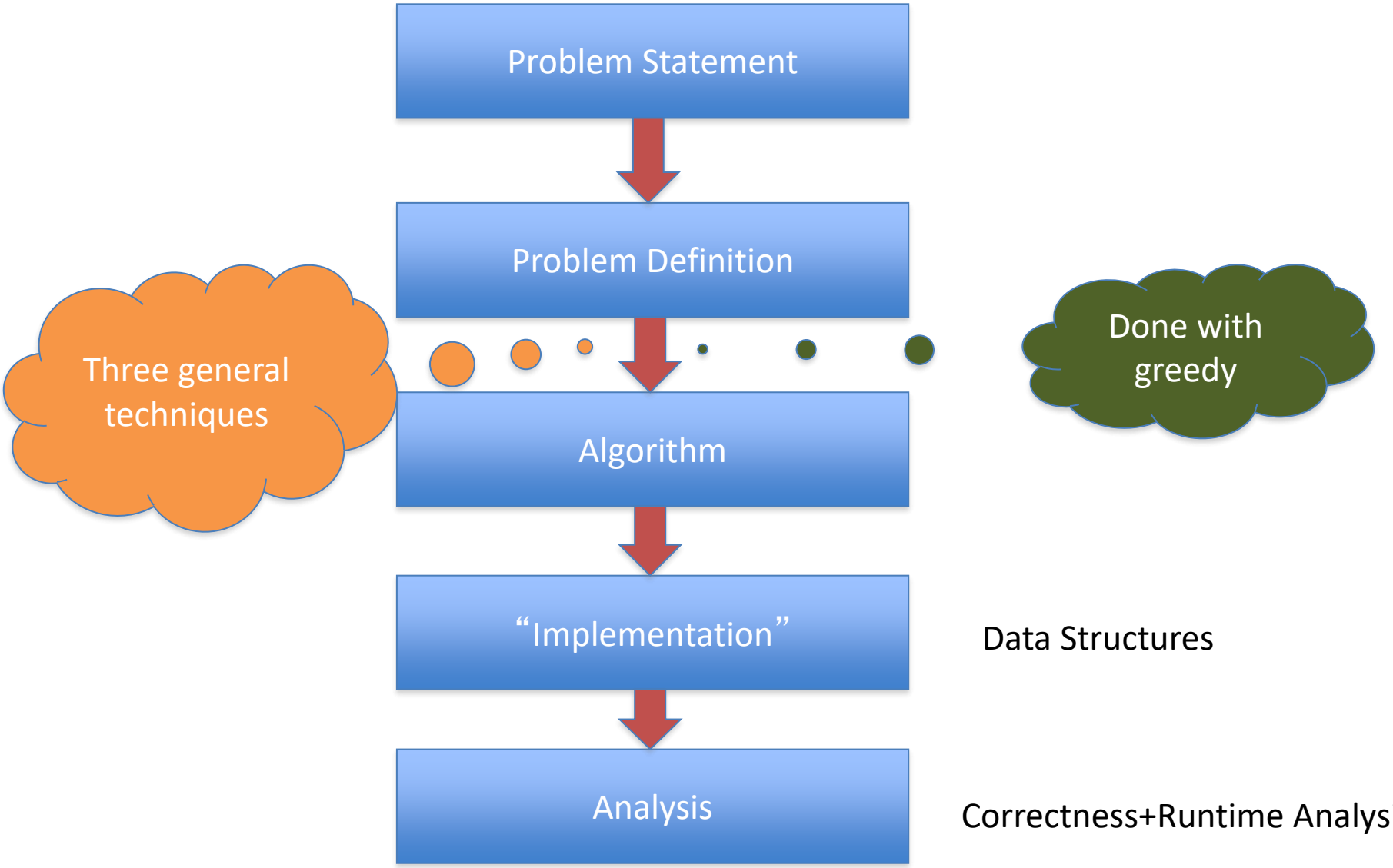
If an edge can be added to T without adding a cycle then add it to T

Can be verified in O(m+n) time

Reading Assignment

Sec 4.5, 4.6 of [KT]

High Level view of the course



Trivia

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

Sorting

Given n numbers order them from smallest to largest

Works for any set of elements on which there is a total order

Insertion Sort

Input: $a_1, a_2, \ldots, a_n$

Output: $b_1, b_2, \ldots, b_n$

$O(n^2)$ overall

Make sure that all the processed numbers are sorted

$b_1 = a_1$

for $i = 2 \ldots n$

Find $1 \le j \le i$ s.t. a_i lies between b_{j-1} and b_j

Move b_j to b_{i-1} one cell "down"

$b_j = a_i$

O(log n)

O(n)

a	b
4	4
3	4
2	4
1	4

Other $O(n^2)$ sorting algorithms

Selection Sort: In every round pick the min among remaining numbers

Bubble sort: The smallest number “bubbles” up

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

Mergesort Algorithm

Divide up the numbers in the middle

Sort each half recursively

Merge the two sorted halves into one sorted output

Unless n=2

How fast can sorted arrays be merged?

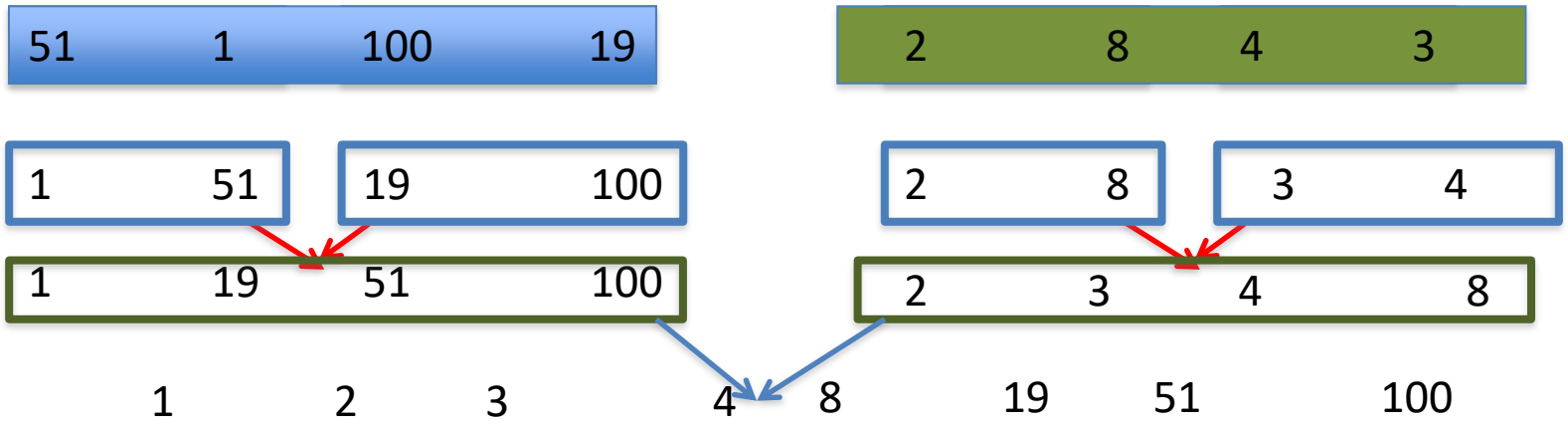
Mergesort algorithm

Input: $a_1, a_2, \ldots, a_n$

Output: Numbers in sorted order

```
MergeSort( a, n )

    If n = 1 return the order a_1
    If n = 2 return the order min(a_1,a_2); max(a_1,a_2)

    a_L = a_1,..., a_n/2

    a_R = a_n/2+1,..., a_n

    return MERGE ( MergeSort(a_L, n/2), MergeSort(a_R, n/2) )
```

An example run



MergeSort(a, n)

If n = 1 **return** the order a_1

If n = 2 **return** the order $\min(a_1,a_2)$; $\max(a_1,a_2)$

$a_L = a_1,\ldots, a_{n/2}$

$a_R = a_{n/2+1},\ldots, a_n$

return MERGE (MergeSort(a_L, n/2), MergeSort(a_R, n/2))

Correctness

Input: $a_1, a_2, \ldots, a_n$

Output: Numbers in sorted order

```
MergeSort( a, n )

  If n = 1 return the order a_1
  If n = 2 return the order min(a_1,a_2); max(a_1,a_2)

  a_L = a_1,..., a_n/2

  a_R = a_n/2+1,..., a_n

  return MERGE ( MergeSort(a_L, n/2) , MergeSort(a_R, n/2) )
```

By induction on n

Inductive step follows from correctness of MERGE

Runtime analysis on the board…