

Lecture 5

CSE 331

Please have a face mask on

Masking requirement



UB requires all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

<https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html>

(Perfect) Matching

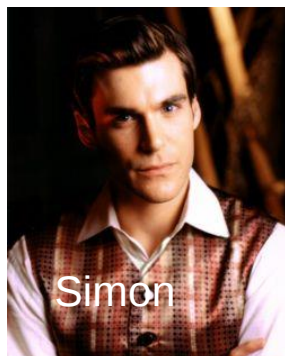
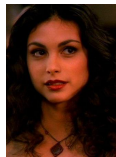
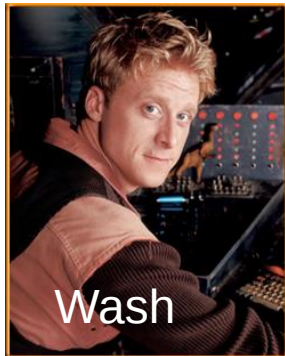
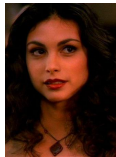
A matching $S \subseteq M \times W$ such that following conditions hold:

S is a **set** of pairs (m,w) where m in M and w in W

- (1) For every woman w in W , exist *exactly* *at most* one m such that (m,w) in S
- (2) For every man m in M , exist *exactly* *at most* one w such that (m,w) in S

Perfect matching

Preferences



Work things out on paper

• A stable marriage

Even though BBT and JA are not very happy



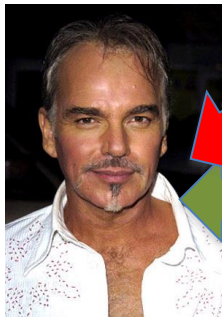
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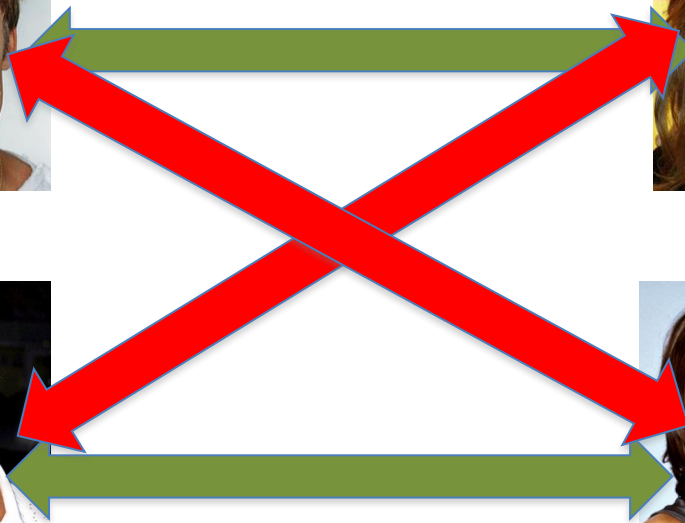
AJ



BBT



JA



Two stable marriages



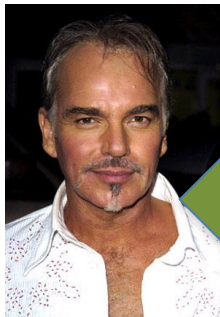
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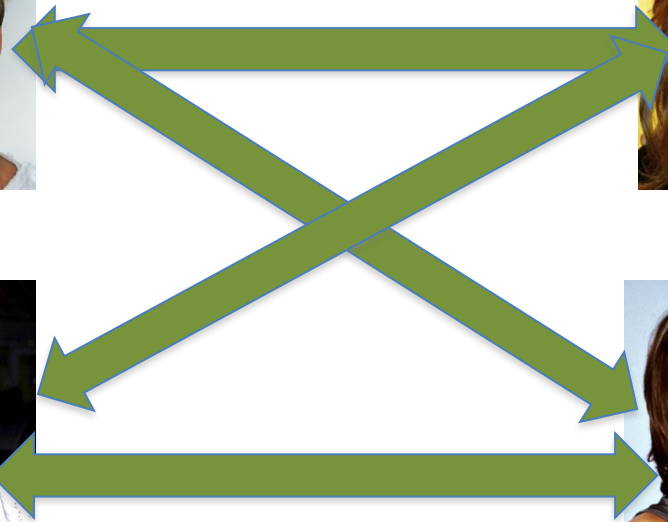
AJ



BBT



JA



Stable Marriage problem

Set of men M and women W

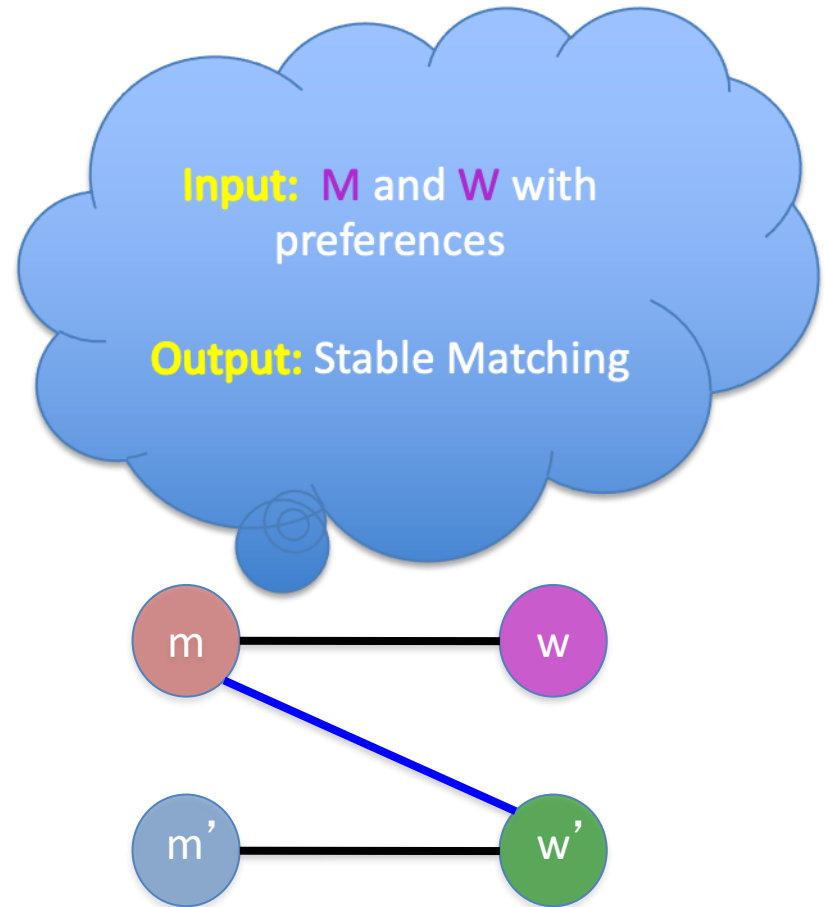
Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in $M \times W$)

Perfect Matching (everyone gets married)

Instability

Stable matching = perfect matching + no instability



Questions/Comments?

Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

The naïve algorithm

Incremental algorithm to produce all $n!$ perfect matchings?

Go through all possible perfect matchings S

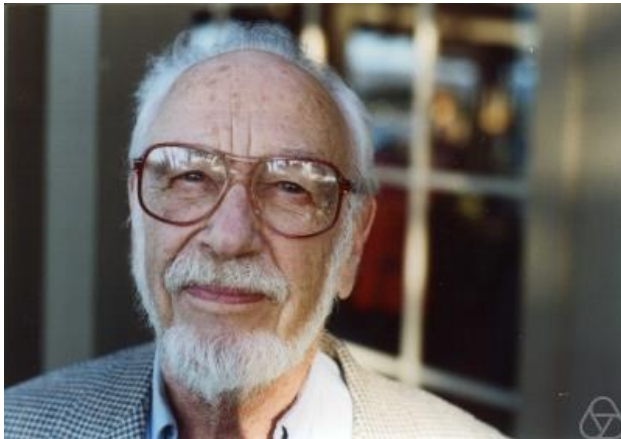
If S is a stable matching

then Stop



Else move to the next perfect matching

Gale-Shapley Algorithm



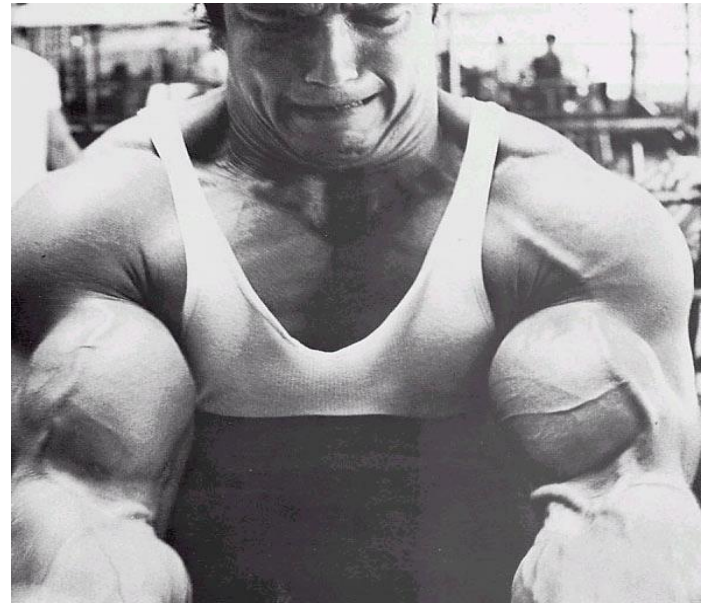
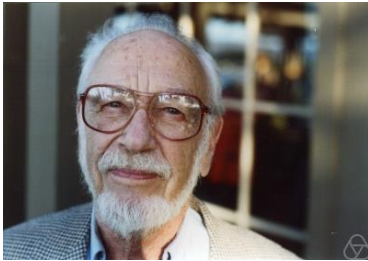
David Gale



Lloyd Shapley

$O(n^2)$ algorithm

Moral of the story...



Questions/Comments?

Rest of today's agenda

Gale Shapley (GS) algorithm

Run of GS algorithm on an instance

Questions/Comments?