Lecture 6

CSE 331

Please have a face mask on

Masking requirement

UB requires all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html

HW 1 gets released Today

Date	Topic	Notes
Week 1 Mon, Jan 31	Introduction F21 F19 F18 F17	Week 1 recitation notes (**HW 0 out**)
Wed, Feb 2	Main Steps in Algorithm Design F21 F19 F18 F17	
Fri, Feb 4	Stable Matching Problem F21 F19 F18 F17 x²	[KT, Sec 1.1]
Week 2 Mon, Feb 7	Perfect Matchings F21 F19 F18 F17 x²	[KT, Sec 1.1] (**HW 0 in**) Week 2 recitation notes
Wed, Feb 9	Algorithms for stable matching problem F21 F19 F18 F17 x²	[KT, Sec 1.1]
Fri, Feb 11	Gale Shapley algorithm F21 F19 F18 F17 x²	[KT, Sec 1.1] (**HW 1 out**) *Reading Assignment:* Pigeonhole principle *Reading Assignment:* Asymptotic notation care package

Reading Assignments

Date	Topic	Notes
Week 1 Mon, Jan 31	Introduction F21 F19 F18 F17	Week 1 recitation notes (**HW 0 out**)
Wed, Feb 2	Main Steps in Algorithm Design F21 F19 F18 F17	
Fri, Feb 4	Stable Matching Problem F21 F19 F18 F17 x²	[KT, Sec 1.1]
Week 2 Mon, Feb 7	Perfect Matchings F21 F19 F18 F17 x²	[KT, Sec 1.1] (**HW 0 in**) Week 2 recitation notes
Wed, Feb 9	Algorithms for stable matching problem F21 F19 F18 F17 x²	[KT, Sec 1.1]
Fri, Feb 11	Gale Shapley algorithm F21 F19 F18 F17 x²	[KT, Sec 1.1] (**HW 1 out**) *Reading Assignment:* Pigeonhole principle *Reading Assignment:* Asymptotic notation care package

Stable Marriage problem

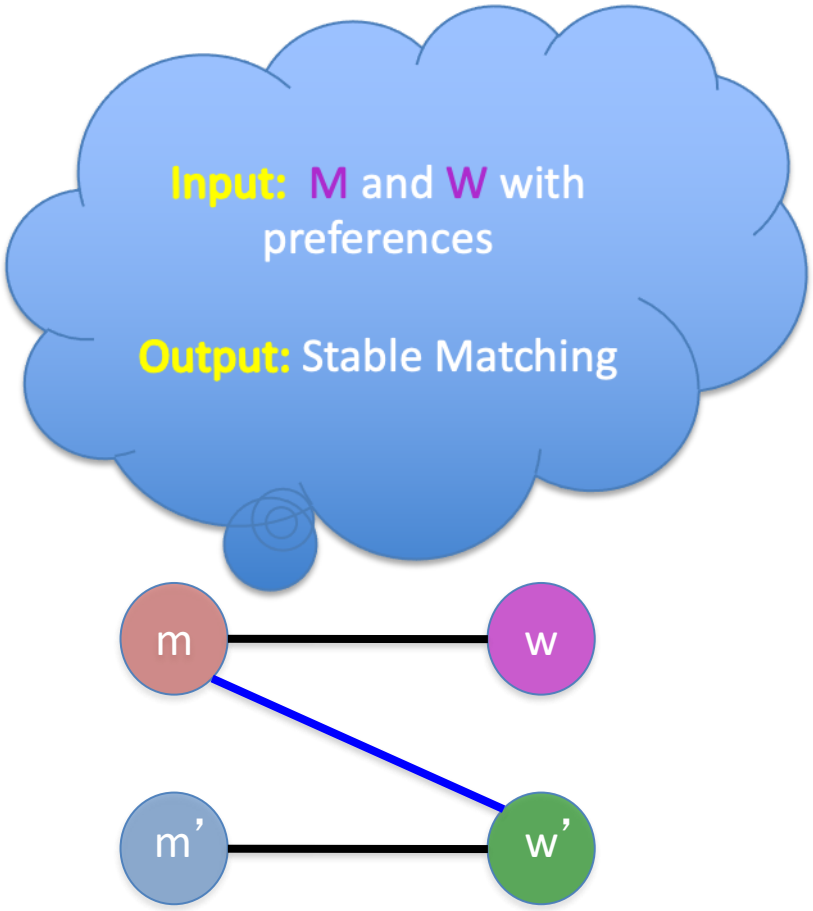
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instablity



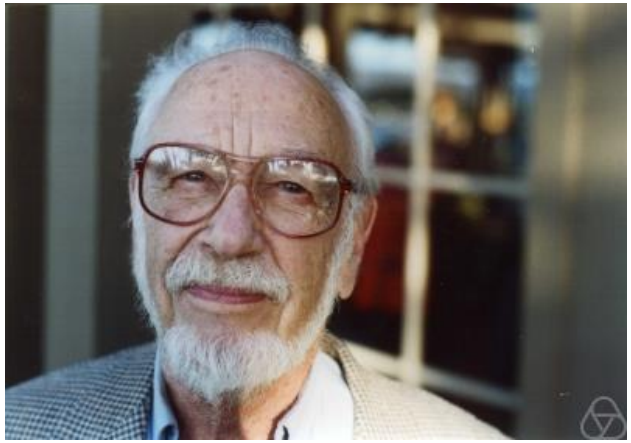
Stable matching = perfect matching+ no instablity

Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

Gale-Shapley Algorithm



David Gale

Lloyd Shapley

$O(n^2)$ algorithm

Gale-Shapley Algorithm

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Output the engaged pairs as the final output

Rest of today's agenda

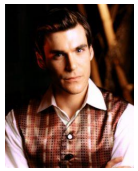
GS algorithm

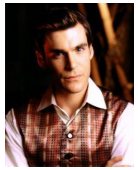
Run of GS algorithm on an instance

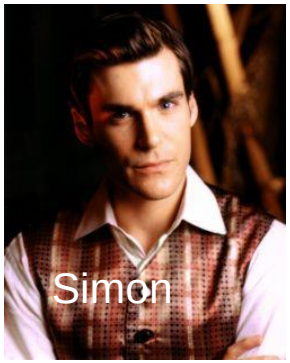
Prove correctness of the GS algorithm

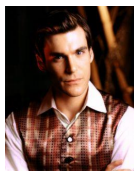
Back to the board…

Preferences









GS algorithm: Firefly Edition

Observation 1

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Once a man gets engaged, he remains engaged (to "better" women)

Output the engaged pairs as the final output

Observation 2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

If w proposes to m after m', then she prefers m' to m

Output the set S of engaged pairs as the final output

Questions/Comments?

Why bother proving correctness?

Consider a variant where any free man **or** free woman can propose

Is this variant any different? Can you prove it?

GS’ does not output a stable marriage