

FOL “Pragmatics” à la Brachman & Levesque

Rather than waste precious class time repeating the definitions that Brachman & Levesque give in their §2.4, I’ll present them here with the kind of commentary I would have given in lecture.

Before launching into this, let me also remind you that I have prepared a document that tries to clarify the satisfaction clause for quantifiers. This document is in PDF at:

<http://www.cse.buffalo.edu/~rapaport/563S05/satisfaction.pdf>

and in HTML at:

<http://www.cse.buffalo.edu/~rapaport/563S05/satisfaction.html>

B&L call the material that they cover in this section “pragmatics”, but I don’t like their use of that term. The meaning of “pragmatics” is controversial. The usual meaning is that pragmatics is a third branch of **semiotics** (i.e., of the theory of signs). The others, you will recall, are **syntax** (which is only concerned with the signs or symbols themselves) and **semantics** (which is concerned with the relationships between the signs and symbols, on the one hand, and the domain that they represent, on the other). In contrast, pragmatics is concerned with the relationships among (1) the signs and symbols, on one hand, (2) the domain that they represent, on the other, **and (3) the user** (on the third hand?). For more information on semiotics, see my webpage “Formal Systems”, at:

<http://www.cse.buffalo.edu/~rapaport/563S05/formalsystems.html>

I prefer the term “semantic inference” for the topics that B&L discuss in this section. Here, then, is my presentation of it (along with a few other topics):

Notation:

1. Let ϕ be a sentence (i.e., a wff with no free variables).

Then we will write:

$$\mathcal{S} \models \phi$$

for: ϕ is true in \mathcal{S} .

(Else we say that ϕ is false in \mathcal{S} .)

2. Let S be a set of sentences.

Then we will write:

$$\mathcal{S} \models S$$

for: $\mathcal{S} \models \phi$, for all $\phi \in S$.

(And we will sometimes say: “ \mathcal{S} is a logical model of S ”.)

Next, there are **semantic** (and, for that matter, syntactic) **relations among** sentences that don’t depend (i.e., these semantic relations don’t depend) on the semantics of the individual sentences. (**Comment 1:** This sounds an awful lot like a denial of the Principle of Compositionality!) **Comment 2:** I suppose that the existence of such intersentential but non-compositional relations is why B&L call this “pragmatics”, because among such relations are phenomena like indexicals (e.g., the meaning of “I am hungry” doesn’t entirely depend on the meanings of its words, but also on who is uttering them). However, the relations that B&L have in mind, as you will soon see, don’t depend on the user.)

Here's an example:

Suppose that: $\mathfrak{S} \models (\alpha \supset \beta)$
and that: $\mathfrak{S} \models \alpha$.
Then: $\mathfrak{S} \models \beta$.

(Looks an awful lot like \supset Elim, or Modus Ponens, doesn't it? But it isn't; rather, it's the semantic analogue of \supset Elim!) This semantic relationship holds no matter what α and β are (i.e., no matter what they "mean"). I.e., the truth of β is **implicit** in the truth of α and $(\alpha \supset \beta)$.

Definitions:

Let S be a set of sentences.

Let α be **any** sentence.

Then:

1. α is a "logical" (better: *semantic*) *consequence* of S
(or: S "logically" (better: *semantically*) *entails* α),
written:

$$S \models \alpha$$

=*df.* for every interpretation \mathfrak{S} , if $\mathfrak{S} \models S$, then $\mathfrak{S} \models \alpha$.

I.e., every **model** of S **satisfies** α .

Note the overloading of the symbol " \models "! Up to now, it has been used as a relation between interpretations (models) and wffs (as in the expression " $\mathfrak{S} \models \alpha$ "). Now it is **also** being used as a relation **among** wffs (as in the expression " $S \models \alpha$ ")!

2. α is "logically" (better: *semantically*) *valid*,
written:

$$\models \alpha$$

=*df.* $\{ \} \models \alpha$.

I.e., $\models \alpha$ iff, for every \mathfrak{S} , $\mathfrak{S} \models \alpha$.

Why have I put the word 'logically' in "scare quotes"? I used the word because that's the word that not only B&L, but almost every other logician, uses. But I have always found it confusing, because I have always viewed logic as a syntactic enterprise. Here, however, we're talking about semantics (or, following B&L, about pragmatics). So I think the term 'semantically' is less confusing. For me, " \vdash " is a syntactic, logical relation. But " \models " is a semantic relation.

Having said that, please note that it is controversial. The philosophical issue is this: Is logic about symbols (syntax) or about their meanings (semantics)? (Or both?) **B&L treat it as if it were about semantics** (or, as they say, "pragmatics"). Their treatment in the rest of the book uses " \vdash ". But what is it that computers do? **They do syntax**, in the sense of symbol manipulation. So, which symbols do they manipulate? The ones of a KR language? Or the ones representing the ontological domain of interpretation? B&L talk as if it were the latter. But I maintain that it is the former.