AN F.O.L. NATURAL-DEDUCTION EXAMPLE

Show $\neg\forall x[\alpha(x)] \vdash \exists x[\neg\alpha(x)]$:

1. $\neg\forall x[\alpha(x)]$ // assumption

/*
   Show $\exists x[\neg\alpha(x)]$:
      Show $\neg\alpha(t)$, for some t & use $\exists$-Intro:
/*

*2. $\alpha(t)$ // temporary assumption (with arbitrary t)
*3. $\forall x[\alpha(x)]$ // 2, $\forall$-Intro
*4. $\neg\forall x[\alpha(x)]$ // send 1
*5. $\neg\alpha(t)$ // 2,3,4, $\neg$-Intro
6. $\neg\alpha(t)$ // return 5
7. $\exists x[\neg\alpha(x)]$ // $\exists$-Intro

Exercise for the reader: Use $\forall$-Intro to show: $\neg\exists x[\alpha(x)] \vdash \forall x[\neg\alpha(x)]$