

UNIFICATION EXAMPLE

Find MGU for $W = \{P(a, x, f(g(y))), P(z, f(z), f(u))\}$.

k	W_k	σ_k	W_k singleton?	D_k	occurs check
0	$W_0 = W$	$\sigma_0 = \{\}$	no	$\{a, z\}$	true; $v_0 = z,$ $t_0 = a$
		$\sigma_1 = \text{Compose}(\{\}, \{a/z\})$ $= \{a/z\}$			
1	$W_1 = \text{Subst}(\{a/z\}, W_0)$ $= \{P(a, x, f(g(y))), P(a, f(a), f(u))\}$		no	$\{x, f(a)\}$	true; $v_1 = x,$ $t_1 = f(a)$
		$\sigma_2 = \text{Compose}(\{a/z\}, \{f(a)/x\})$ $= \{a/z, f(a)/x\}$			
2	$W_2 = \text{Subst}(\{f(a)/x\}, W_1)$ $= \{P(a, f(a), f(g(y))), P(a, f(a), f(u))\}$		no	$\{g(y), u\}$	true; $v_2 = u,$ $t_2 = g(y)$
		$\sigma_3 = \text{Compose}(\{a/z, f(a)/x\}, \{g(y)/u\})$ $= \{a/z, f(a)/x, g(y)/u\}$			
3	$W_3 = \text{Subst}(\{g(y)/u\}, W_2)$ $= \{P(a, f(a), f(g(y))), P(a, f(a), f(g(y)))\}$ $= \{P(a, f(a), f(g(y)))\}$		yes		

Returns $\sigma_3 = \{a/z, f(a)/x, g(y)/u\}$ as MGU,
with $P(a, f(a), f(g(y)))$ as the common instance.