Robert C. Moore, "Reasoning about Knowledge and Action", *IJCAI-77*: 223–227.

Logical Axioms (Definitions):

Def $[[T(w,\alpha)]]$ = The object-language wff $[[\alpha]]$ is true in possible world $[[w]]$.

L1 $\text{True}(\alpha) \equiv T(w_0,\alpha)$

L2 $T(w,(\alpha \text{ And } \beta)) \equiv (T(w,\alpha) \wedge T(w,\beta))$

L3 $T(w,(\alpha \text{ Or } \beta)) \equiv (T(w,\alpha) \vee T(w,\beta))$

L4 $T(w,(\alpha \Rightarrow \beta)) \equiv (T(w,\alpha) \supset T(w,\beta))$

L5 $T(w,(\alpha \Leftrightarrow \beta)) \equiv (T(w,\alpha) \equiv T(w,\beta))$

L6 $T(w,\text{Not}(\alpha)) \equiv \neg T(w,\alpha)$

Definitions & Axioms for Knowledge:

Def $[[K(a,w,w')]] = [[w']]$ is a world that is possible according to what agent $[[a]]$ knows in world $[[w]]$.

K1 $T(w,\text{Know}(a,\alpha)) \equiv \forall w'[K(a,w,w') \supset T(w',\alpha)]$

K2 $K(a,w,w)$

K3 $K(a,w,w') \supset [K(a,w',w'') \supset K(a,w,w'')]$

K4 $K(a,w,w') \supset [K(a,w,w'') \supset K(a,w',w'')]$

- $\therefore$ K is an equivalence relation for fixed a;
  $\therefore$ this is an S5 modal (epistemic) logic.

Definitional Axioms for Quantifiers:

L7 $T(w,\text{Exist}(v,\alpha(v))) \equiv \exists x[T(w,\alpha(x/v))]$, for x not free in α.

L8 $T(w,\text{All}(v,\alpha(v))) \equiv \forall x[T(w,\alpha(x/v))]$, for x not free in α.

L9 $T(w,\text{Eq}(t_1,t_2)) \equiv (t_1 = t_2)$

Definitional Axioms for Results of Actions:

Def $[[\text{Res}(e,\alpha)]]$ = it is possible for event $[[e]]$ to occur, & wff $[[\alpha]]$ would be true in the Resulting situation.

Def $[[R(e,w,w')]] = [[w']]$ is a possible world that could result from event $[[e]]$ occurring in world $[[w]]$.

Def $[[\text{Do}(a,c)]]$ = the event consisting of agent $[[a]]$ Doing command $[[c]]$.

R1 $T(w,\text{Res}(e,\alpha)) \equiv (\exists w'[R(e,w,w')] \wedge \forall w'[R(e,w,w') \supset T(w',\alpha)])$

R2 $T(w,\text{Res}(\text{Do}(a,\text{Loop}(\alpha,c)),\beta)) \equiv T(w,\text{Res}(\text{Do}(a,\text{If}(\alpha,(c;\ \text{Loop}(\alpha,c)),\text{Nil})),\beta))$

R3 $T(w,\text{Res}(\text{Do}(a,\text{If}(\alpha,c,c')),\beta)) \equiv ([T(w,\text{Know}(a,\alpha)) \wedge T(w,\text{Res}(\text{Do}(a,c),\beta))]$
$\vee [T(w,\text{Know}(a,\text{Not}(\alpha))) \wedge T(w,\text{Res}(\text{Do}(a,c'),\beta))])$

R4 $T(w,\text{Res}(\text{Do}(a,(c;\ c')),\alpha)) \equiv T(w,\text{Res}(\text{Do}(a,c),\text{Res}(\text{Do}(a,c'),\alpha)))$

N1 $R(\text{Do}(a,\text{Nil}),w,w') \equiv (w = w')$

Definition of "Can"

C1 $T(w, \text{Can}(a, \alpha)) \equiv \exists c[T(w, \text{Know}(a, \text{Res}(\text{Do}(a, c), \alpha)))]$

> **Note:** The English words 'can' and 'know' are *etymologically* related in exactly this way! You "can" do something iff you "ken"—i.e., know—how to do it. From *American Heritage Dictionary of the English Language*, at dictionary.com: 'can' comes from "Middle English, first and third person sing. present tense of `connen`, *to know how*."

Frame Axioms (Definitions) for Dialing Combinations of Safes

D1 $\exists w'[R(\text{Do}(a, \text{Dial}(x_1, x_2)), w, w')] \equiv [T(w, \text{Comb}(x_1)) \wedge T(w, \text{Safe}(x_2)) \wedge T(w, \text{At}(a, x_2))]$

D2 $$R(\text{Do}(a, \text{Dial}(x_1, x_2)), w, w') \supset \bullet\ [T(w, \text{Is-comb-of}(x_1, x_2)) \supset T(w', \text{Open}(x_2))]$$
$$\wedge [(\neg T(w, \text{Is-comb-of}(x_1, x_2)) \wedge \neg T(w, \text{Open}(x_2)) \supset \neg T(w', \text{Open}(x_2)))]$$
$$\wedge [T(w, \text{Open}(x_2)) \supset T(w', \text{Open}(x_2))]$$

D3 $$R(\text{Do}(a, \text{Dial}(x_1, x_2)), w, w') \supset :\ [K(a, w', w'') \equiv \bullet$$
$$[(T(w', \text{Open}(x_2)) \equiv T(w'', \text{Open}(x_2)))$$
$$\wedge \exists w'''[K(a, w, w''') \wedge R(\text{Do}(a, \text{Dial}(x_1, x_2)), w''', w'')]]]$$

Facts about Combinations

A1 $T(w, \text{Is-comb-of}(x_1, x_2)) \supset [T(w, \text{Comb}(x_1)) \wedge T(w, \text{Safe}(x_2))]$

A2 $T(w, \text{At}(a, x)) \supset T(w, \text{Know}(a, \text{At}(a, x)))$

The Problem

Given: True(At(John, safe_1)) $\wedge$ True(Exists(X_1, Know(John, Is-comb-of(X_1, safe_1))))

Prove: True(Can(John, Open(safe_1)))

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