

A complete language for EXPSPACE:
PIM, "Polynomial Ideal Membership"—the simplest natural completeness level that is known not to have polynomial-size circuits.

Succinct 3SAT

$n \times n$ Chess

For any fixed k , there is a problem in this intersection that can NOT be solved by circuits of size $O(n^k)$

SAT

NLIN

NTIME $[n^2]$

NP

P

DLIN

etc

DTIME $[n^3]$

DTIME $[n^2]$

PIM

EXPSPACE

NEXP

co-NEXP

EXP

QBF

PSPACE

Σ_2^P

Π_2^P

NP

FACT

P

NL

L

REG

REG \neq L

TAUT

co-NP

CVP

GAP

UGAP

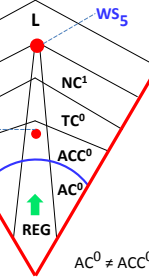
WS₅

Unknown but Commonly Believed:

- $L \neq NL \dots \dots \dots L \neq PH$
- $P \neq NP \cap co-NP \dots \dots P \neq PSPACE$
- $NP \neq \Sigma_2^P \cap \Pi_2^P \dots \dots NP \neq EXP$

Best Known Separations:

- $AC^0 \subset ACC^0 \subset PP$, also $TC^0 \subset PP$
- $NC^1 \subset PSPACE, \dots, NL \subset PSPACE$
- $P \subset EXP, NP \subset NEXP$
- $PSPACE \subset EXPSPACE$



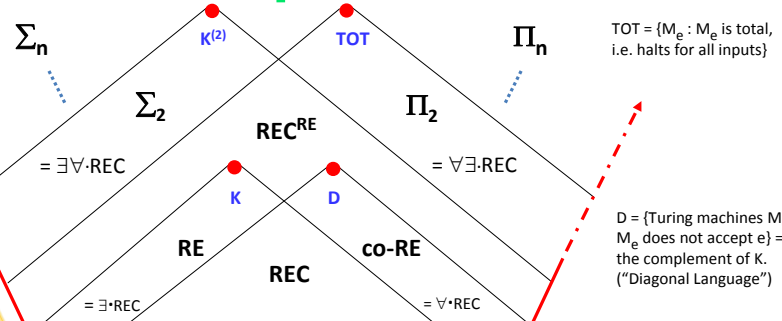
C Low-Level Classes

WS₅, the word problem for the symmetric group S₅, is a regular language that is complete for NC¹ under AC⁰ many-one reductions.

B Complexity "Main Sequence"

A Deterministic and Nondeterministic Time Hierarchies Within NP

Arithmetical Hierarchy (AH)



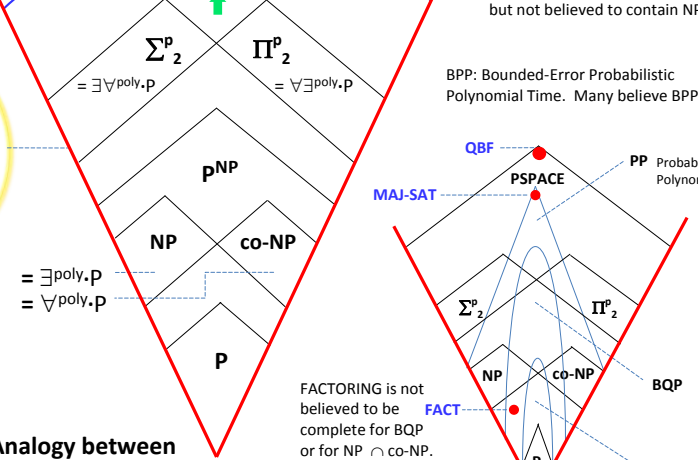
TOT = {M_e : M_e is total, i.e. halts for all inputs}

D = {Turing machines M_e : M_e does not accept e} = the complement of K. ("Diagonal Language")

BQP: Bounded-Error Quantum Polynomial Time. Believed larger than P since it has FACTORING, but not believed to contain NP.

BPP: Bounded-Error Probabilistic Polynomial Time. Many believe BPP = P.

Polynomial Hierarchy (PH)



The levels of AH and PH are analogous, except that we believe $NP \cap co-NP \neq P$ and $\Sigma_2^P \cap \Pi_2^P \neq P^{NP}$, which stand in contrast to $RE \cap co-RE = REC$ and $\Sigma_2 \cap \Pi_2 = REC^{RE}$

FACTORING is not believed to be complete for BQP or for $NP \cap co-NP$.

D Analogy between Arithmetical and Polynomial Hierarchies

E Realm of Feasibility?

A Deterministic and Nondeterministic Time Hierarchies Within NP