

Games With Oracles That Lie

Joint work with Richard J. Lipton, Georgia Tech

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A Classic “Liar Puzzle”

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 - If he says ‘no’ then he truthfully would say no if you asked directly, so Left goes to D and Right goes to W.

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- 2 Now suppose the man is a Knave:
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So regardless of whether the answer is a lie, if you hear ‘yes’ you should go Left, else go Right.

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 - If telling the truth, then it is true that he could say 'yes.' But the latter could be his allowed lie...

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 - Chess is **Hard** to play. How hard?

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- Question is “really about” **Fault-Tolerance** in computing.

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- Ask K : What move should Dragon play at Q ?

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- Starting with an initial position P with legal moves m_1, \dots, m_ℓ and the premise—or promise—that P is W , focus on playing an optimal move from P .
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- Hence if you *win* the hypo-play, you know m_i was not a lie and can *commit* to it.

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- Whew.
- When the basic idea for the $k = 1$ case won't fit on one Beamer frame, you know the whole thing is complicated. . .

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- Further work on algorithms to improve these bounds is in progress. . . END for now, thanks!